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ABSTRACT

The purpose of this project is to teach learning and understanding of mathematics at grades seven through nine through the use of science experiments. Previous knowledge of science on the part of students or teachers is not necessary. Lists of needed equipment are found at the beginning of this volume. It is strongly recommended that teachers try each experiment before it is done in class. The material in this part of the program can be covered in four weeks. The material in this book is used to develop the concepts of negative numbers, the basic properties of the real number system, linear functions, and quadratic functions. Included in the Teacher's Commentary are background information, discussion of activities and exercises, and answers to problems. (RH)

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SCHOOL
MATHEMATICS
STUDY GROUP

MATHEMATICS THROUGH SCIENCE

PART II: GRAPHING, EQUATIONS AND LINEAR FUNCTIONS

TEACHERS' COMMENTARY

(revised edition)

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MATHEMATICS THROUGH SCIENCE

Part II: Graphing, Equations and Linear Functions

Teachers' Commentary

(revised edition)

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FOREWORD

During the summer of 1963, a group of fifteen mathematicians, scientists, and teachers, working under the auspices of the School Mathematics Study Group, prepared experimental textbook units which explored the possibility of developing some of the basic concepts of mathematics through simple, but significant, experiments in physical science. These units were tested in representative classrooms in a number of centers over the country during the following school year. In the summer of 1964, revisions were made on the basis of the results of these trials.

The purpose of this project was to see if the learning and understanding of mathematics at the grade levels seven through nine could be improved by approaching mathematical principles through science experiments. The results of the preliminary tests were quite encouraging. The students found the experiments fascinating and learned the related mathematical principles quickly and easily. Many suggestions for improvements were made by teachers on the basis of the reactions of their students. These suggestions were incorporated into this revision wherever possible.

It should be noted that these units were written for use in the mathematics classroom and that they are primarily designed to teach mathematical concepts rather than those of science. It is true that procedures and principles of science included are sound and correct within the framework in which they are used. The experience in science which the student will gain from the study of these units will no doubt be useful in subsequent courses in the physical sciences, but the main purposes of the units are to teach mathematics.

Previous knowledge of science on the part of students or teachers is not necessary, although any knowledge which they might have in this field will be useful. In particular, it is not necessary for the student to have studied the earlier parts of this work. The material in this book is complete in itself.

The experimental procedures are clearly described. Lists of needed equipment will be found at the beginning of this volume. Many substitutions are possible in these lists since the experimental equipment is not at all critical. The teacher is warned, however, that there is a natural perversity in experimental work. It is strongly recommended that the teacher try out each experiment before it is done by the class, particularly if equipment substitutions must be made.

For best results, the students should do the experiments themselves or in small groups. A teacher demonstration should be resorted to only when absolutely necessary. Even when this is done, every effort should be made to involve the students as much as possible.

The material in this textbook can be covered in about four weeks. The teacher can best decide at what time in the course its use might be most appropriate. These units supplement, but do not replace, whatever mathematics book is in regular use in the classroom. It may, however, be possible to omit some sections of the regular textbook if the teacher feels that the treatment of the corresponding topics in this text is sufficient.

The experiments in this book are used to develop the concepts of negative numbers, the basic properties of the real number system, linear functions, and quadratic functions. The teacher should look over the contents of the chapters and skip or cover only briefly any material which has already been learned by the students. On the other hand, any work done in this text need not be repeated in the regular part of the course. This will probably include work on linear functions, quadratic functions and graphing.

Even if the earlier chapters are skipped or covered only briefly, Chapter 3 should be covered. It is in this chapter that the student will learn how to handle experimental data. The remaining chapters will then show him how to analyze the data which occurs in more complicated situations.

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PREFACE

Most of the mathematical techniques that are in use today were developed to meet practical needs. The elementary arithmetic operations have obvious uses in everyday life, but the mathematical concepts which are introduced at the junior high school level and above are not as obviously useful.

The School Mathematics Study Group has been exploring the possibility of introducing some of the basic concepts of mathematics through the use of some simple science experiments. Several units were prepared during the summer of 1963 and were used on an experimental basis in a number of classrooms during the following year. On the basis of the results of these trials, these units were revised during the summer of 1964.

The text is designed to be usable with any mathematics textbook in common use. It is not meant to replace the textbook for the course, but to supplement it.

Previous acquaintance with science on the part of the student is unnecessary. The scientific principles involved are fairly simple and are explained as much as is necessary in the text. Each experiment opens a door into a new domain in mathematics: the real number system, linear functions, quadratic functions. We hope that student learning and understanding will be improved through the use of this material.

The experiments have all been done in actual classroom situations. Every effort has been made to make the directions for the experiments as clear and simple as possible. The apparatus has been kept to a minimum.

The writers sincerely hope that this approach to mathematics will prove both useful and interesting to the student.

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EQUIPMENT LIST

Part II

Sources for equipment in the following experiments are indicated below and are coded at the right of each item.

(1) Scientific supply (i.e., Cenco or Welch, etc.)

(2) Hardware store

(3) Stationery store

(4) Variety store

(5) Home

Chapter 1

OPEN SENTENCES AND EQUATIONS

1. The Seesaw Experiment - students are to work in groups. Each group should have the following equipment:

   1 meter stick (1) or yardstick (2)

   1 triangular file (5 in) extra-slim taper - (2), or 1 knitting needle - (4)

   1 paper clip spring-type (Hunt Brand 1) - (3)

   1 box 1 "Gem" paper clips - (3)

   1 4" x 4" x 6" block of wood - (5)

   4 6-ounce "Dixie" cups - (4)

   2 dowels, 12 inches long - (2)

   1 lump modeling clay - (4)

   1 set of standard weights (10, 20, 20, 50, 200, 200 grams) - (1)

   1 piece of strong string or nylon thread - (4)

Chapter 2

AN EXPERIMENTAL APPROACH TO LINEAR FUNCTIONS

1. The Loaded Beam - students are to work in groups. Each group should have the following equipment:

   1 15-inch flexible wooden ruler - (4)

   1 3-inch C clamp - (2)

   1 meter stick - (1), or yardstick - (2)

   1 spool button thread - (4) (for entire class)

   1 set of hooked weights (10, 20, 20, 50, 100, 200 grams) - (1)

2. The Falling Sphere Experiment - students are to work in groups. Each group should have the following equipment:

   1 glass cylinder or jar at least 8 inches high

   1 steel ball bearing, about $\frac{1}{8}$ inch diameter - (bicycle shop)

   1 small magnet - (2)

   1 12-inch ruler, also calibrated in centimeters - (3)

   1 bottle Karo Syrup (white) - (grocery)

   4 paper strips, about 1" × 10" - (5)

   1 roll cellophane tape - (4) for entire class

   1 metronome - (school)

Chapter 3

TRAMPOLINES AND GASES

1. The Trampoline Experiment - This experiment is a teacher demonstration.

   1 9-inch aluminum pie plate - (4)

   6 15-cent balloons, spherical - (4)

   1 10 × 24-inch sheet of bristol board - (3)

   1 pound of plastolene - (4)

   2 glass marbles - (4), or $\frac{5}{8}$-inch nylon bearings - (bearing supply)

   1 desk lamp (or slide projector) - (school)

2. Gay-Lussac's Law - This experiment is a teacher demonstration.

   1 Gas Law apparatus - (1), Cenco No. 76407 (Chem study apparatus)

   2 thermometers, Centigrade (-20° to 110°)

   1 electric hot plate - (5) or (school)

   2 trays of ice cubes

   32 sheets ($8\frac{1}{2}$" × 11") onion-skin paper - (3), or frosted acetate - (engineering supply)

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Chapter 1
EQUATIONS AND OPEN SENTENCES

1.1 Introduction

There is a two-fold purpose in Chapter 1. First, to achieve a greater understanding of equations and open sentences, and to achieve a measure of competence in their solution. Second, to learn, through example as well as precept, the importance of the mathematics involved in solving problems evolved through scientific experimentation.

The experiments contained in this chapter have been tried by 8th graders. It was found that the students' involvement in the actual experimentation is very important. It is suggested, therefore, that a teacher demonstration would be a poor second choice. The student needs to develop scientific intuition and a spirit of inquiry. There are many ways of learning. Students need to explore. They need to actually manipulate the physical apparatus of an experiment. For example, in the Balanced Meter Stick Experiment the student can physically feel the balance of the meter stick.

1.2 The Seesaw Experiment (the Balanced Meter Stick)

The Balanced Meter Stick Experiment requires the following material:

- 1 meter stick
- 1 paper clamp #1 (hunt brand or similar type)
- 1 5" triangular file, extra-slim taper.
- 1 4" x 4" x 6" block of wood
- 4 6-ounce "Dixie" cups
- 2 dowels, 12 inches long
- 1 lump of modeling clay
- 1 set of standard weights (10, 20, 20, 50, 100, 200 grams)
- 1 piece of strong string or nylon thread
- 1 box #1 paper clips

Place a paper clamp at the 50 cm mark of a meter stick. Be precise, for it is necessary that the center of the clamp be at the midpoint of the meter stick. A small triagular file is run through the hole in the clip so that half the file's length is on either side of the clip. An edge of the file should be upright. A 20-penny nail or a knitting needle could be used instead of the file.

Now, place two Dixie cups upside down on a block of wood sufficiently far apart to allow the ends of the file to rest over the outer edges of the inverted cups. The meter stick will swing between the two cups.

On the table near the ends of the meter stick place chunks of clay close behind the meter stick. Stand the dowel in the clay in an upright position. When the meter stick is in a horizontal position, mark the dowel. These marks will then be used as reference marks for the horizontal position of the meter stick. It is necessary to have these marks drawn accurately for they will be used to indicate the horizontal position of the meter stick throughout the experiment.

The paper clip hangers can be made easily by bending paper clips as illustrated in Figure 1.

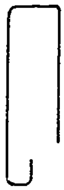

Figure 1

Masses can be readily hung on paper clips bent in this manner. In addition, the clips will slide easily on the meter stick.

In making up a meter stick model of a seesaw, feel free to use your ingenuity and to make whatever changes are helpful, according to the composition of the class and the materials available at the time.

Coke bottles can be used to replace the block and Dixie cups. The meter stick must be high enough that the attached masses will not strike the table when the stick is not quite in balance.

Also, the meter stick can be supported on a knife edge by means of a knife edge clamp, commercially made for this purpose. (See Figure 2.)

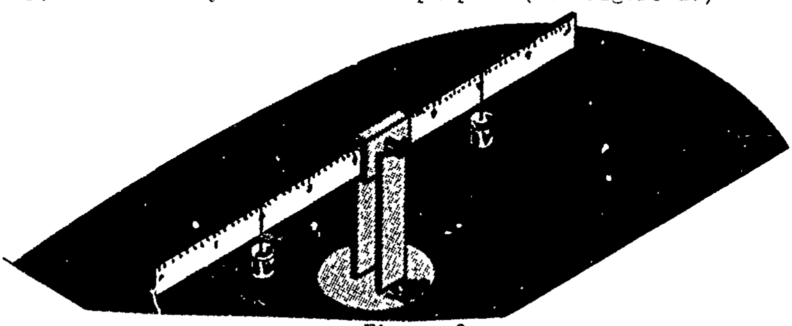

Figure 2

The teacher should use this commercially made knife edge with care since it is very sensitive, especially for masses greater than 1 0 grams. The sensitivity problem can be minimized by attaching a piece of modeling clay at the bottom part of the clamp screw. (See Figure 3.)

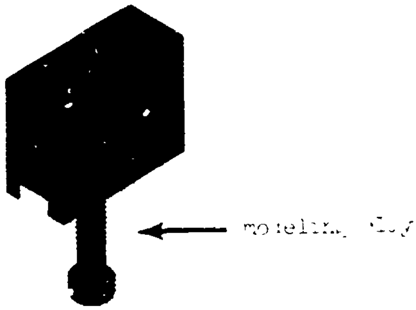

Figure 3

If the standard masses cannot be obtained you can use pennies (or washers) instead. The pennies can be suspended from the stick by small plastic bags. Instead of grams, the mass measure would be expressed in terms of the pennies. For instance, the mass measure of the object could be equivalent to the mass measure of 25 pennies. A second alternative can be lead fishing weights. These are stamped with a number representing its weight in ounces. The split type of weights can be clamped directly onto the bent paper clips.

Before starting the actual experiment, the students should have some preliminary play on balancing, just to get the feel of it. It is important that the stick settle in a horizontal position before the distance is read off. The use of two dowels can help appreciably in this respect. The 6-ounce paper cup at each end of the stick is used to eliminate extreme tipping of the stick. (See Figure 4)

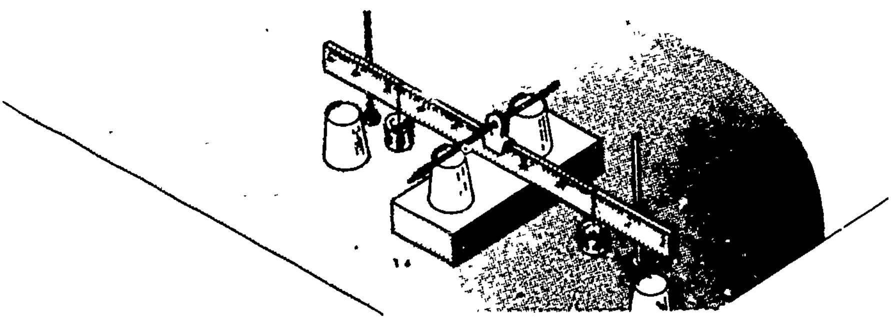

Figure 4

The distance should be estimated to the nearest centimeter.

Finding a general rule from the observations can be an exciting experience for the students. The teacher should use his imagination to lead the students to the discovery of the general rule: md = 1200.

Exercise 1

1. Below is a table of values from an experiment with a seesaw. Masses were hung on the right-hand side to balance the 6 pounds at 8 cm from the fulcrum on the left side. Find where we should place the masses shown in the table on the right side of the seesaw to balance it with 6 pounds placed at 8 cm from the fulcrum on the other side.

Left side		Right side	
m	d	Mass of objects in pounds m	Distance from the fulcrum in cm d
6	8	12	4
6	8	2	24
6	8	8	6
6	8	24	2
6	8	16	3
6	8	6	8

2. Find the values for the masses and distance in the given table if you want to balance 20 gm at 14 cm from the fulcrum on the other side.

m gm	20	40	10	$18\frac{2}{3}$	$9\frac{1}{3}$	50
d cm	14	7	28	15	30	$5\frac{3}{5}$

3. How far from the fulcrum should a 20-gm mass be placed on the left side to balance a 40-gm mass placed 20 cm from the fulcrum on the right side?

    40 cm

4. A boy, whose mass is 70 lbs, rode a seesaw with his father, whose mass is 175 lbs. If the father sat 4 ft from the fulcrum, where must the boy sit to balance the seesaw?

    10 ft

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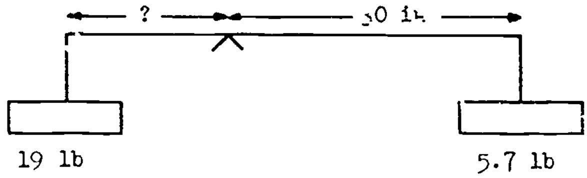
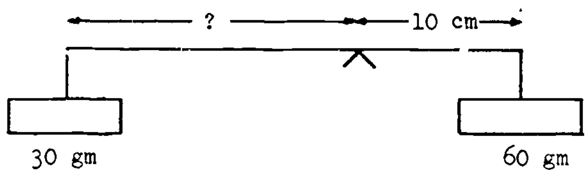
Find the missing values in problems 5, 6 and 7.

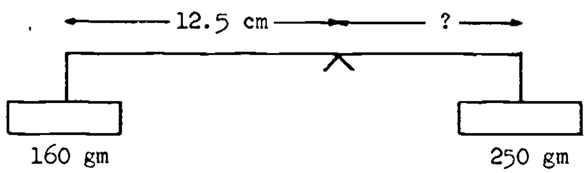
5. 9 in



6. 20 cm



7. 8 cm



8. Is there a place on the seesaw where a single mass can be placed and a balance obtained? If so, what is the distance of the mass from the fulcrum?

   At the 50 cm mark. (This would be true for any mass.) The mass is at zero distance from the fulcrum.

1.3 Number Sentences

In this and several subsequent sections the components needed for the understanding and solution of equations and inequalities will be presented. The student will be led through the topics that build toward the open sentence, and, later, to the solution of sentences. This chapter is not intended to give the student complete mastery in the solution of equations, but will provide an opportunity for the student to gain a degree of facility in such solutions. The student should also gain an understanding of the processes that lead to such solutions.

As in English, mathematics makes wide use of sentences. Sentences are used both to explain and to discuss mathematics. The mathematical sentences that make statements about numbers is the subject of this section.

Number sentences, in themselves, may be equalities (as $2 + 7 = 12 - 3$), or inequalities (as $5 > 4$); they may be true (as $15 + 3 < 26 - 1$) or false (as $8 + 9 = 24 + 3$). However, just as with English sentences, when we write a sentence the assumption usually is that we are asserting its truth.

1.4 Number Phrases

One of the components of a number sentence is the number phrase. A number phrase is basically a name for a number. The student must recognize that a number may have many names. Therefore, many number phrases may be written concerning a given number.

This section offers the student an opportunity to express number phrases in mathematical terms involving both letter and number.

In translating number phrases into mathematical symbols the student must take great care. For example, he must be capable of distinguishing between "5 less than the number x", written $x - 5$, and "5 is less than the number x", written $5 < x$. As another example, consider the differences between "7 plus x", written $7 + x$; "7 more than x", written $x + 7$; or "7 greater than x", written $7 > x$. The need for such distinctions are frequent in mathematics, and require precision and exactness of expression.

Exercise 2

1. Translate each of the following number phrases into mathematical symbols.

(a) The sum of the number x and 15. $x + 15$

(b) The product of 8 and x. $8x$

(c) One fourth of the number x. $\frac{1}{4}x$ or $\frac{x}{4}$

(d) A number which is 4 less than x. $x - 4$

(e) The division of 18 by x. $\frac{18}{x}$

(f) Three greater than x. $x + 3$

(g) One less than two thirds of x. $\frac{2}{3}x - 1$

(h) The number x less than 23. $23 - x$

2. For each of the number phrases in Problem 1 find the number represented by the phrase if $x = 12$.

(a) $12 + 15 = 27$

(b) $8(12) = 96$

(c) $\frac{12}{4} = 3$

(d) $12 - 4 = 8$

(e) $\frac{18}{12} = \frac{3}{2}$ or $1\frac{1}{2}$

(f) $12 + 3 = 15$

(g) $\frac{2}{3}(12) - 1 = 7$

(h) $23 - 12 = 11$

1.5 Parentheses

Parentheses are used to set number phrases apart so that confusion in the order of various operations is unlikely to occur. The parentheses is one of the signs used in mathematics to indicate an aggregate of mathematical terms. When several mathematical terms are placed within parentheses, the whole is treated as a single term. For example, when translated into symbolic form $(12 + 18 - 5)$ is treated as a single number. This would be equally true of $(x + y - a)$.

Exercise 3

1. Which of the following closed phrases name the same number? a, b, e

(a) $2 + 4 \times 5$ and 22 — $2 + 20 = 22$

(b) $(2 + 4)5$ and 30 — $(6)5 = 30$

(c) $2 + (4 \times 5)$ and 30 — $2 + (20) \neq 30$

(d) $4 + 3 \times 2$ and $(4 + 3)2$ — $4 + 6 \neq (7)2$

(e) $5 \times 8 + 3$ and $(5 \cdot 8) + 3$ — $40 + 3 = (40) + 3$

(f) $32 - 8 - 4$ and $6 \times 4 + 5$ — $4 - 4 \neq 24 + 5$

2. Place parentheses in the following so that

(a) $2 \times 3 + 1$ represents 8 — $2 \times (3 + 1)$

(b) $2 + 4 \times 3$ represents 14 — $2 + (4 \times 3)$

(c) $6 \times 3 - 1$ represents 17 — $(6 \times 3) - 1$

(d) $12 - 1 \times 2$ represents 22 — $(12 - 1) \times 2$

(e) $18 - 6 \div 3$ represents 16 — $18 - (6 \div 3)$

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3. Find a number for each numerical phrase.

(a) $5 \times 8 + 7$ — $40 + 7 = 47$

(b) $5(8 + 7)$ — $5(15) = 75$

(c) $(9 + 1)(3 + 4)$ — $(10)(7) = 70$

(d) $6 + 2 \cdot 4$ — $6 + 8 = 14$

(e) $14 - 3 \times 2$ — $14 - 6 = 8$

(f) $(17 - 6)4$ — $(11)4 = 44$

(g) $(\frac{6 + 2}{4}) + 5$ — $\frac{8}{4} + 5 = 7$

(h) $9(1 + 3) - (8 + 2)$ — $9(4) - 10 = 26$

(i) $9(1 + 3) - 8 + 2$ — $9(4) + 2 - 8 = 30$

4. Using parentheses, rewrite the following closed phrases so they represent the same number. For instance, $2 \times 5 + 6 \times 2$ can be written $(2 \times 5) + (6 \times 2)$ and both represent 22.

(a) $3 + 8 - 4$ — $(3 + 8) - 4$ or $3 + (8 - 4)$

(b) $\frac{1}{2} \times 6 + 4$ — $(\frac{1}{2} \times 6) + 4$

(c) $3 \times 5 - 4 \times 2$ — $(3 \times 5) - (4 \times 2)$

(d) $36 \div 9 + 5 - 2$ — $(36 \div 9) + (5 - 2)$

1.6 Distributive Property of Numbers

In the distributive property the student first confronts a property involving two operations. As stated in the text, the students are working with the distributive property for multiplication over addition. For convenience, in this text we shall simply refer to this as the distributive property.

This property may be illustrated to the student very simply. Using any desired objects on a table or marks on a blackboard, place them in 2 rows of 3 each and 2 rows of 7 each. For example,

```
***    *******
***    *******
```

If the asterisks were moved together, there would be 2 rows of 10 each. Now find the value of the 3 parts,

2×3, 2×7, and 2×10 as 6, 14, and 20.

In the distributive form it appears as

$$\begin{aligned}(2 \times 3) + (2 \times 7) &= 2(3 + 7)\\ &= 2(10).\end{aligned}$$

This general type of illustration should be repeated until the students understand the meaning of the distributive property.

The distributive property operates equally well when the number of terms is extended. We may write $5(a + b + c)$. The application of the distributive property eliminates the need for grouping within the parentheses. Students may verify this in the following manner:

$$5(a + b + c) = 5(a + (b + c)) \ .$$

By the distributive property

$$5(a + (b + c)) = 5a + 5(b + c) \ .$$

Therefore,

$$5(a + b + c) = 5a + 5b + 5c \ .$$

Exercise 4

1. Which of the following problems are indicated sums and which are indicated products?

   (a) $3(8 + 5)$ (d) $4(3 + 6)$

   (b) $3(8) + 3(5)$ (e) $7 + (3 \times 6)$

   (c) $2(6) + 2(3)$ (f) $(7 + 3)6$

   The indicated sums are b, c and e. The indicated products are a, d and f.

2. Express the following indicated products as indicated sums and indicated sums as indicated products.

   (a) $4(47 + 3)$ $4(47) + 4(3)$

   (b) $9(34 + 6)$ $9(34) + 9(6)$

   (c) $\frac{2}{3}(8) + \frac{2}{3}(4)$ $\frac{2}{3}(8 + 4)$

   (d) $18(3.2) + 18(.8)$ $18(3.2 + .8)$

3. Perform the indicated operations the easier way. Show your method.

   Illustrative Example:

   $$\begin{aligned} &110(8) + 110(92) \\ &= 110(8 + 92) \\ &= 110(100) \\ &= 11,000 \end{aligned} \qquad \text{or} \qquad \begin{aligned} &110(8) + 110(92) \\ &= 880 + 10120 \\ &= 11,000 \end{aligned}$$

   (a) $12(\frac{1}{3} + \frac{1}{4})$ $= 12(\frac{1}{3}) + 12(\frac{1}{4})$

   $= 4 + 3$

   $= 7$

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(b) $\frac{1}{5}(\frac{7}{8}) + \frac{1}{5}(\frac{1}{8})$ $= \frac{1}{5}(\frac{7}{8} + \frac{1}{8})$

$= \frac{1}{5}(1)$

$= \frac{1}{5}$

(c) $9(11 + 9)$ $= 9(20)$

$= 180$

(d) $0(17 + 83)$ $= 0$

(e) $\frac{8}{9}(0 + 9)$ $= \frac{8}{9}(9)$

$= 8$

4. Show how you could use the distributive property to perform the multiplications mentally.

Example: $6 \times 24 = 6(20 + 4)$

$= 6(20) + 6(4)$

$= 120 + 24$

$= 144$

(a) $7(22)$ $= 7(20 + 2)$

$= 140 + 14$

$= 154$

(b) $12(33)$ $= 12(30 + 3)$

$= 360 + 36$

$= 396$

(c) $15(36)$ $= 15(30 + 6)$

$= 450 + 90$

$= 540$

1.7 Translation of Open Phrases to Word Phrases

Translating open phrases into word phrases requires both care and skill. An open phrase may be translated into many different word phrases.

Each translation should be written out in full. There is a danger that shortcuts and abbreviations may lead to incorrect thought patterns. Therefore, students should be required to make complete translations.

Once the thought pattern required in making the translation has been developed there should be little difficulty in completing the work of this section.

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Exercise 5

The answers to this exercise will differ widely and their correctness must be verified by the teacher:

1. Can you think of a different way to translate the phrase $2x + 3$ into a word phrase?

2. How many translations of $3x - 5$ can be made? Give examples.

3. In the following problems, write a translation of the phrase to a verbal phrase.

(a) $n + 6$	(e) $\frac{n}{3}$
(b) $n - 6$	(f) $\frac{n + 1}{3}$
(c) $2n$	(g) $x + 7x$
(d) $2n + 1$	(h) $2x + 3x$

1.8 Translation of Word Phrases to Open Phrases

It is imperative that students develop a capability to translate word phrases into mathematical terms, that is, into open phrases. It is necessary that the student be able to state explicitly what the letters stand for whenever they are used in an open phrase. For example, the meaning of the open phrase $x + 7$ cannot be fully determined until the meaning of x has been explicitly stated.

Guard against such statements as

"x = John",

when students are expressing the meaning of the symbols. Make sure that they understand that the symbol represents a number. The meaning of the symbol must be explicitly stated. For example,

x = John's age in years

f = the length of the barn in feet.

Vary the symbols used. The symbol x should be used occasionally, but the symbols a, n, t and others should also be used.

Since the commutative property is not valid for subtraction, a word phrase such as "the difference between 3 and 5" may not have any valid translation. Should this be translated $5 - 3$ or $3 - 5$? Because of the ambiguity in translation, such word phrases must be avoided. There must be special caution taken to avoid generalizations in word phrases which involve the operations of subtraction and division since these operations are not commutative.

Students must have sufficient practice in the translations involved in this section to achieve the measure of competence needed for success in courses in mathematics beyond the 8th grade. It is because of the importance of this work that Exercise 6 has been made rather lengthy.

Exercise 6

1. Translate the following word phrases to symbols.

(a) If the number of years in Bill's age is now K, what is the number of years in Bill's age 7 years from now? $K + 7$

(b) The numb... cents in x quarters $25x$

(c) The number of cents in x dollars $100x$

(d) The number of years in Sam's age 3 years ago $\underline{x - 3}$, where x represents Sam's present age in years.

(e) The number of years in John's age 4 years from now $\underline{a + 4}$, if a represents John's present age in years.

(f) The number of feet in y yards $3y$

(g) The number of inches in b yards $36b$

2. Translate each of the following word phrases to symbolic phrases.

(a) The sum of a number x and 2 $x + 2$

(b) The number x decreased by 8 $x - 8$

(c) The number x subtracted from 15 $15 - x$

(d) The product of 7 and x $7x$

(e) The quotient of a number 3 divided by x $\frac{3}{x}$

(f) The number x increased by 6 $x + 6$

(g) The number x divided by 2 $\frac{x}{2}$

(h) One third of a number x $\frac{1}{3}x$

3. For each of the number phrases in Problem 1, find the number represented by the phrase if the unknown number is 24.

(a) $24 + 7 = 31$
(b) $25(24) = 600$
(c) $100(24) = 2{,}400$
(d) $24 - 3 = 21$
(e) $24 + 4 = 28$
(f) $3(24) = 72$
(g) $36(24) = 864$

4. Write open phrases to represent each of the following.

(a) The sum of an even number and the next even number

$x + (x + 2)$, if x is an even number

(b) One half of the sum of a number and 6

$\frac{x + 6}{2}$ or $(x + 6)\frac{1}{2}$

(c) Seven less than 3 times a number $3x - 7$

(d) Twice a number increased by 3 $2x + 3$

(e) Twice the sum of 7 and 2 $2(7 + 2)$

(f) Find the total age of Mary and Sue if Mary is 5 times as old as Sue is.
[Hint: let x represent the number of years in Sue's age.] $x + 5x$

(g) The number of cents Mike has, if he has x nickels and twice as many dimes as nickels

$5x + 2(10x)$ or $5x + 20x$

5. If the sum of the number t and 3 is doubled, which of the following phrases would be a correct name for the sum:

$2t + 3$ or $2(t + 3)$? $2(t + 3)$

6. If 5 is added to twice a certain number n and the sum is divided by 3, which phrase is the correct name for the quotient:

$\frac{2n + 5}{3}$ or $\frac{2n}{3} + 5$? $\frac{2n + 5}{3}$

7. If one fourth of a certain number x is added to one third of four times the same number, which phrase is the correct name:

$\frac{1}{3}(4x) + \frac{1}{4}(x)$ or $\frac{4}{3}(x) + \frac{1}{4}(x)$? $\frac{1}{3}(4x) + \frac{1}{4}(x)$

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8. If the number of gallons of milk purchased is y, which is the correct phrase for the number of quart bottles that will contain it:

$$4y \quad \text{or} \quad \frac{y}{4} \ ?$$

4y

9. If a is the number of feet in the length of a certain rectangle and b is the number of feet in the width of the same rectangle, which phrase is the correct name for the perimeter:

$$2(a + b) \quad \text{or} \quad ab \ ?$$

2(a + b)

Fill in the blanks in the foll wing problems.

10. If k represents a number of kilometers, then the phrase 1000k represents the number of meters in k kilometers.

11. A mathematical phrase indicating the number of centimeters in s meters is 100s.

12. Given a symbol d representing the number of liters in a container, the phrase 1000d represents the number of milliliters in that container.

13. The number of grams in p milligrams is $\frac{1}{1000}p$.

14. The number of grams in t kilograms is 1000t.

15. Therefore the sum of t kilograms and w grams would be ((1000t + w) g).

16 The number of centimeters in k meters and n centimeters would be ((100k + n) cm).

17. Adding t centigrams to s grams would result in a sum of $((\frac{1}{1000}t + s)\ g)$.

18. In a mixture made up of oxygen and nitrogen, there are 4 times as many oxygen molecules as nitrogen molecules. Write a mathematical phrase for the number of oxygen molecules if there are h molecules of nitrogen. 4h

1.9 Numerical Sentences

The purpose of this section is to introduce the numerical sentence and its associated terminology. Numerical sentences are formed by connecting number phrases by verb phrases, often using the equal sign. When phrases are written and connected by an equal sign (=) we make the assumption that the sentence is true. But it is perfectly reasonable to talk about a false sentence. However, in the course of operations or proofs, it is unreasonable to use false sentences.

When applied to sentences, the words "true" and "false" are preferable to "right" and "wrong" or "correct" and "incorrect" because the latter two sets imply moral judgments without mathematical basis. The student should be encouraged to use only "true" and "false" in this context.

Exercise 7

Indicate whether each of the following sentences is true or false.

1.	(a)	$3(15) = 3(10) + 3(5)$	True
	(b)	$4(8) + 4(2) = 4(10)$	True
	(c)	$4(2 + 3) = 4(2) + 5$	False
	(d)	$13 + (4 \times 5) = (13 + 4)5$	False
	(e)	$4 \times 6 + 3 = 5 \times 7 - 8$	True
	(f)	$3 + 4 \times 5 - 9 = 2 \times 6 + 7$	False
	(g)	$14 - 8 + 6 \times 8 = 18 \times 7 - 23$	False
	(h)	$31 \times 23 - 42 \times 7 = (27 \times 8) + 7(16)$	False

1.10 Open Sentences

In Section 1.9 numerical sentences emphasized verb phrases, and no trouble arose in deciding whether such a sentence was true or false. In Section 1.10, the concept of sentence is enlarged to "open sentences" which involve variables.

An open sentence involving one variable has a "truth set" defined as the set of numbers for which the sentence is true. There is no need at this time to introduce a name for the set which makes a sentence false. The term "solution set" is also used for "truth set", particularly for sentences which are in the form of equations. In the solution of open sentences the concept of the "truth set" or "solution set" becomes very important.

A phrase or sentence is "open" in the sense that we cannot determine precisely what number is named by it until we know what numbers are named by the variables.

Finally, the student is asked to consider both equations and inequalities as sentences. These topics are of equal interest and have equal right to the students' attention.

Exercise 8

1. In the following problems assume that the domain of the variable is the set of all real numbers. Use your knowledge of arithmetic to find the solution set for each of the open sentences.

   (a) $x + 3 = 5$ — $x = 2$

   (b) $y + 3 > 5$ — $y > 2$

   (c) $4x = 12$ — $x = 3$

   (d) $4x \neq 12$ — set of all real numbers except 3

   (e) $\frac{n}{6} = 2$ — $n = 12$

   (f) $b + 8 < 10$ — $b < 2$

2. Replace the box with a number that will make the sentence true.

   (a) $\square + 3 = 12$ — 9

   (b) $4 + \square = 8$ — 4

   (c) $3 \times \square + 2 = 23$ — 7

   (d) $4 \times \square = 20$ — 5

3. In each of the following examples, select those elements of the domain which make the open sentence true.

   (a) $x + 2 = 12$ — $\{8, 4, 6, 10\}$ is the domain of x — 10

   (b) $3x = 12$ — $\{6, 2, 4\}$ is the domain of x — 4

   (c) $16 - y = 10$ — $\{8, 10, 6\}$ is the domain of y — 6

   (d) $x^2 + 4 = 8$ — $\{0, 2, 4\}$ is the domain of x — 2

4. Let n represent the number of people that go to the local movie on Saturday night. What is the domain of n? If all tickets cost \$1.35 each, and the total collection for one night is \$236.25, how many people bought tickets?

   Domain of n is the set of positive integers.

   $\$1.35\, n = \$236.25$

   $n = 175$

5. Let g represent the number of gallons of gasoline you buy at the filling station. What is the domain of g? If each gallon costs 30¢ and you pay $2.76, now many gallons did you buy?

   The domain of g is the set of all positive rational numbers.

   $\$.30\ n = \$2.76$

   $n = 9.2$ gallons .

6. Let p represent the number of people who go to a dance at which only 30 couples are admitted. What is the domain of p? If each couple must be accompanied by a chaperon, what is the domain of p?

   (a) Domain of p is the set of positive numbers less than 61:

   (b) Domain of p is the set of positive numbers less than 91.

1.11 Equations and Inequalities

In this section open sentences are separated into two classes, and given their common names: equations and inequalities. It must be emphasized again that the student and teacher consider both equations and inequalities on an equal basis. Both are equally interesting and useful, and of equal importance.

Exercise 9

Express in equation form the following:

1. Assume the cost of gasoline is 32¢ per gallon, and C represents the total cost of gasoline in cents. Write an equation for the total cost of n gallons of gasoline. $C = 32n$

2. Write an equation for the cost d in dollars of n gallons of gasoline at 32¢ per gallon. $d = .32n$

Write in symbolic form the following statements:

3. The diameter (D) of a circle equals twice the radius (r). $D = 2r$

4. The perimeter (P) of a triangle equals the sum of its sides (a, b and c). $P = a + b + c$

5. Which of the following sentences are true and which are false?

(a)	$5 + (8 + 3) = (5 + 8) + 3$	True
(b)	$6 + 4 \neq 2(4 + 1)$	False
(c)	$5 + 2 = 3 + 4$	True
(d)	$3.5 - 2.9 \nless 2.3$	False
(e)	$8(3) \neq 3(8)$	False
(f)	$\frac{1}{3} + \frac{1}{4} > \frac{1}{2}$	True

Answers to Problems 6 and 7 will vary widely. The correctness of the solution is left to the teacher.

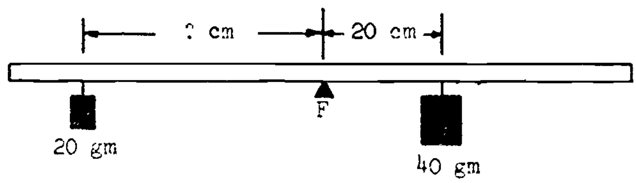
6. Write five <u>true</u> sentences involving each of the symbols: $<, >, \nless, \ngtr, \neq$.

7. Write five <u>false</u> sentences involving each of the symbols in Problem 6.

8. Put a numeral in place of the symbol $\square$ so that the sentence in each case will be true.

(a)	$\square + 1 = 7$	6
(b)	$\square - 3 \neq 5$	any number except 8
(c)	$3 \times \square = 12$	4
(d)	$6 \div \square < 2$	any number greater than 3 or less than 0

9. How far from the fulcrum should you place a 20-gm weight on the left side to balance a 40-gm weight 20 cm from the fulcrum on the right side of the stick? (See illustration.)



40 cm

10. How far from the fulcrum should you place a 20-gram mass on the left side of the stick to get the following inequality:

$20 \text{ gm} \times \underline{d} \text{ cm} < 40 \text{ gm} \times 2 \text{ cm}$? $\quad$ $d < 4$ cm

To get $20 \text{ gm} \times \underline{d} \text{ cm} > 40 \text{ gm} \times 2 \text{ cm}$? $\quad$ $d > 4$ cm

Can you get more than one answer?

Yes, as indicated by the answers given in the form of inequalities.

1.12 Finding Unknown Masses by Experiment

In Section 1.2 the student performed an experiment to establish an equation. By reading off the distances on the meter stick and knowing the masses he established that an equality resulted when the meter stick balanced. The seesaw can be extended, however, to lead naturally to multiplicative inverses and the solution of equations by this method.

In the experiments in this section one of the masses is unknown. It should truly be an unknown. Rocks are suggested for convenience only. Other masses of suitable size will serve equally well. Variations in the type of masses to be measured can heighten the students' interest. For example, enclose the unknown mass in a sealed box and disclose the nature of the contents only when the correct mass has been determined. Guessing the nature of the object from the mass alone can be a further extension.

The exercise following this section is important to the student. It gives him experience in the solution of equations without the confusion of terminology.

1.13 Multiplicative Inverse

Although the student uses the multiplicative inverse in Section 1.12, it is not so named. The student should realize, however, that in the solution of equations the use of the multiplicative inverse is quite general.

The depth to which the multiplicative inverse is to be developed may depend on the background and personnel of the class. The multiplicative inverse is needed as a manipulative technique for future chapters of the text. However, manipulation must be based on understanding. Therefore, it is important that the student understand the concept of the multiplicative inverse.

It should be sufficient to introduce the identity element for multiplication which is 1. Multiplying any number by 1 does not change the number. That is, it remains identically the same number. This can be expressed in symbols as:

$$1 \times n = n \times 1 = n .$$

From this the definition of inverse can follow:

if c and d are numbers

and $\qquad cd = 1$

then d is called the multiplicative inverse of c.

Conversely, it follows from properties of numbers that c is the multiplicative inverse of d.

The multiplicative properties of zero indicate that there is no multiplicative inverse of zero. The product of zero and any number is zero.

The generalization states that one number is the inverse of the other if their product is one.

There has been no mention of the additive inverse since it is not an outgrowth of the experiment. Also, this concept would require an understanding of negative numbers which have not as yet been introduced.

Exercise 10

Find the multiplicative inverse of each of the following numbers.

1. (a) 17 $\frac{1}{17}$ (e) $.6$ $\frac{10}{6}$ (i) $\dfrac{1}{\frac{2}{3}}$ $\frac{3}{2}$

(b) $\frac{8}{7}$ $\frac{7}{8}$ (f) x $\frac{1}{x}$ (j) $a + 1$ $\frac{1}{a + 1}$

(c) $\frac{4}{5}$ $\frac{5}{4}$ (g) $\frac{7}{3}$ $\frac{3}{7}$ (k) $5\frac{1}{2}$ $\frac{2}{11}$

(d) 1 1 (h) 4 $\frac{1}{4}$ (l) $\frac{1}{(ice)^3}$ $(ice)^3$

1.14 Solving Equations

In Section 1.12 the student was introduced to the solution of equations. Further practice is provided using the concepts of the multiplicative inverse. It should be emphasized that solving an equation means reducing the equation so that "x" stands alone. This is accomplished by a series of operations which generate new equations equivalent to the preceding one. The student should not be allowed to omit steps, but should follow in detail the examples given in the text. Only when he understands the method thoroughly can he be allowed to take shortcuts.

The first sentence with "x" alone defines the "solution" or "truth" set. In the cases considered in this chapter, the solution set has only one member.

<u>Exercise 11</u>

1. Solve the following by use of the multiplicative inverse.

(a) $12x = 6$

$12 \cdot \frac{1}{12}x = 6 \cdot \frac{1}{12}$

$x = \frac{1}{2}$

(b) $7x = 14$

$x = 2$

(c) $\frac{8}{7}x = 56$

$x = 49$

(d) $15 = \frac{5}{3}y$

$y = 9$

(e) $5y = 2$

$y = \frac{2}{5}$

(f) $\frac{7}{3}x = 1$

$x = \frac{3}{7}$

(g) $\frac{2}{3}a = \frac{2}{3}$

$a = 1$

(h) $10x = \frac{8}{5}$

$x = \frac{4}{25}$

(i) $2.3y = 4.6$

$y = 2$

2. Translate each of the following sentences into symbols and then solve the equation for the unknown.

(a) The number x multiplied by 5 is equal to 30.

$5x = 30, \quad x = 6$

(b) When a number y is divided by 4 the quotient is 9.

$\frac{y}{4} = 9, \quad y = 36$

(c) The product of $\frac{2}{7}$ and the number a is 28.

$\frac{2}{7}a = 28, \quad a = 98$

(d) Jane bought x stamps for 3¢ each. How many stamps did she buy if she paid 60¢ altogether?

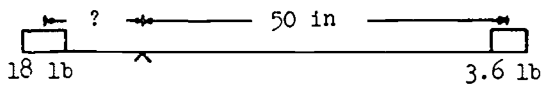
$3x = 60, \quad x = 20$

(e) How old is Susan if 9 times her age is 63?

$9x = 63, \quad x = 7$

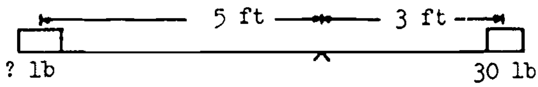
3. Find the missing values in each case.

(a)



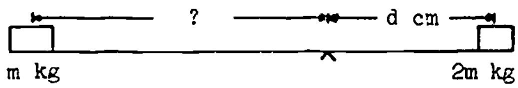
10 in

(b)



18 lb

(c)



2d cm

4. Do you suppose a 90-pound girl could ever lift a 1000-pound box? Justify your answer.

   Yes, if the girl is $\frac{1000}{90}$ times as far as the box from the fulcrum and is on the opposite side.

5. A child whose weight is 60 pounds asked his father, whose weight is 180 pounds, to ride a seesaw with him. Where should the father sit to balance the child if she sits 6 feet from the fulcrum?

   2 feet from the fulcrum

6. A bar 6 feet long is being used as a lever to lift a stone. What is the weight of the stone if a boy weighing 100 pounds pushing down on one end of the bar which is 4 feet from the fulcrum just balances the stone on the other end?

   200 lbs

Sample Test Items

1. A 100-lb boy wishes to lift a 1000-lb block. If he places a fulcrum one foot from the block, where can he pull (how far from the fulcrum) to lift the block?

2. Find a number for each numerical phrase.

   (a) $5(6 + 3)$

   (b) $(9 + 1)(2 + 7)$

   (c) $7(1 + 5) - (6 + 1)$

   (d) $\left(\frac{5 + 2}{2}\right) + 5$

3. Use the distributive property of multiplication to solve:

   (a) $12(33)$

   (b) $(8 \times 3) + (8 \times 2)$

   (c) $(\frac{1}{2} \times 7) - (\frac{1}{2} \cdot 1)$

4. Translate the following word phrases into open phrases.

(a) The sum of a number x and 2.

(b) One third of a number x.

(c) One half the sum of a number x and 6.

(d) The number of centimeters in x meters.

(e) Three less than twice a number b.

5. Which of the following sentences are true and which are false?

(a) $\frac{11}{12} < \frac{12}{13}$

(b) $13 + (4 \times 5) = (13 + 4)5$

(c) $6 + 4 \neq 2(4 + 1)$

(d) $3.5 - 2.9 > 2.3$

(e) $\frac{1}{3} + \frac{1}{4} > \frac{1}{2}$

6. A child whose mass is 50 lb asked his father, whose mass is 175 lb, to ride a seesaw with him. Where should the father sit to balance the child if he sits 7 feet from the fulcrum?

7. Place the parentheses in the following so that:

(a) $2 + 4 \times 3$ represents 14

(b) $5 \times 2 - 5$ represents -15

(c) $8 \times 2 + 3$ represents 19

(d) $10 - 6 \times 2$ represents -2

(e) $5 + 4 \times 9$ represents 81

8. Solve the following for x by the use of the multiplicative inverse.

(a) $\frac{9}{2}x = 27$

(b) $\frac{1}{4}(8x) = 16$

(c) $\frac{1}{2}(3 + 2) = \frac{1}{2}x$

(d) $\frac{3}{7}x = 9(7 - 6)$

9. A rock is balanced 30 cm from the fulcrum opposite a 100-gm mass which is 40 cm from the fulcrum. What is the mass of the rock?

10. Translate the following problems into symbols and then solve the equation for the unknown.

   (a) The number x multiplied by 4 is equal to 24.

   (b) When a number y is divided by 6 the quotient is 24.

   (c) The product of 0.7 and the number a is 19.6 .

   (d) Three times the sum of x and 6 equals 24.

Answers to Sample Test Items

1. Any distance greater than 10 ft.

2. (a) 45
   (b) 90
   (c) 35
   (d) $8\frac{1}{2}$

3. (a) 12(10 + 3)
   (b) 8 × (2 + 3)
   (c) $\frac{1}{2}$ (7 - 1)

4. (a) x + 2
   (b) $\frac{x}{3}$ or $\frac{1}{3}$ x
   (c) $\frac{x + 6}{2}$
   (d) 100x
   (e) 2b - 3

5. (a) True
   (b) False
   (c) False
   (d) False
   (e) True

6. 2 feet from the fulcrum.
   9 feet from the boy.

7. (a) 2 + (4 × 3)
   (b) 5 × (2 - 5)
   (c) (8 × 2) + 3
   (d) 10 - (6 × 2)
   (e) (5 + 4) × 9

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8. (a) 6
   (b) 8
   (c) 5
   (d) 21

9. 75 gm

10. (a) $4x = 24;\ x = 6$
    (b) $\frac{y}{6} = 24;\ \ y = 144$
    (c) $0.7a = 19.6;\ \ a = 28$
    (d) $3(x + 6) = 24;\ \ x = 2$

Chapter 2

AN EXPERIMENTAL APPROACH TO LINEAR FUNCTIONS

2.1 Introduction

The purpose of this section of the book is to teach your students some of the fundamental concepts of mathematics regarding linear functions. These are expressions which involve variables to the first degree. Linear functions are actually first degree polynomials in a single variable.

The need for such knowledge can be presented to the students through the physical sciences. As we tell the students, "The experimental situation will lead to the development of the particular mathematics which is needed". However, after the mathematics is introduced in this manner, a number of logical extensions will be made.

This approach to mathematics must not involve telling the student what will take place. The student himself must encounter first-hand the experimental situations from which the mathematics will arise. He must do the experiments himself, measure the things which change, record the data in an orderly fashion and examine it critically for whatever general relations it shows.

You will be given the results of the Loaded Beam Experiment performed by the writing team. The data is listed in Table 1. However, the student is expected to perform the experiment, find his own data, and make his own analysis. Students may work in teams, and the data obtained by each team may differ because of differences in the rulers and the way they are clamped. But if their work has been done carefully, these differences should not be too great.

In this chapter two experiments are used. Each results in a linear function, but illustrates different properties of this function. The first experiment, The Loaded Beam, investigates what happens when a beam (a 15-inch flexible ruler) is clamped to a desk at one end and loaded with a series of increasing masses at the free end. This gives a series of ordered pairs of numbers which are graphed on a coordinate plane. Through discussion, the student is led to the conclusion that, if other masses had been used, many other ordered pairs could have been found and graphed within the interval of the experiment. This leads finally to a "best straight line" which will, within the errors of measurement, be a graphical representation between the two variables: - load on the end of the beam and the resulting deflection of the end of the beam. At this point the mathematics of the slope and intercept of the line is introduced. This is followed by a discussion of graphing linear functions and an introduction to relations and functions. A similar procedure will be used in each of the experiments.

The experiment on The Falling Sphere will also be concerned with linear functions. However, this experiment will introduce the additional concept of the point-slope form of a linear equation.

The Loaded Beam Experiment

The equipment needed for each student group for the Loaded Beam Experiment is as follows:

1 15-inch flexible wooden ruler
1 3-inch C clamp
1 meter stick or yardstick
1 spool button thread
1 set of hooked weights to include 1 10-gram, 2 20-gram, 1 50-gram, 1 100-gram and 1 200-gram weights.

As in all experiments, the student should be cautioned to exercise care in obtaining data in this experiment. The ruler must be clamped securely to the desk. The student should make a note of the distance his ruler extends beyond the desk top. This will help to insure that all trials in the experiment will be carried on under the same conditions. Also, all problems which subsequently arise from the experiment will relate to consistent data. Furthermore, should a trial need to be rerun, the same conditions could be reproduced.

A hole of about $\frac{1}{8}$-inch diameter should be drilled in the ruler about 1 inch from the end of the ruler. The button thread (or fish line) is looped around the ruler and drawn through the hole. The thread must be sufficiently strong to support a mass of 300 grams. The weights will be hung on a loop the student makes in the thread under the ruler.

The meter stick should be taped to another student desk. The meter stick must rest firmly on the floor. Masking tape will hold it to the desk.

The student is now ready to attach the hooked weights to the ruler and read the deflection on the meter stick.

The student groups should perform the experiment at least three times using the following sequence of loads: 0, 30, 60, 90, 120, 150, 180, 210, 240, 270 and 300 gm. Table 1 shows sample data taken from four runs.

THE LOADED BEAM EXPERIMENT				
Load l (grams)	Trial 1 Position p (centimeters)	Trial 2 Position p (centimeters)	Trial 3 Position p (centimeters)	Trial 4 Position p (centimeters)
0	20.0	20.1	20.1	20.2
30	20.4	20.5	20.4	20.6
60	20.8	20.9	20.9	21.0
90	21.3	21.4	21.4	21.5
120	21.7	21.9	21.8	21.9
150	22.1	22.3	22.3	22.3
180	22.5	22.7	22.8	22.7
210	22.9	23.2	23.2	23.1
240	23.4	23.6	23.7	23.6
270	23.8	24.0	24.2	24.0
300	24.2	24.4	24.6	24.4

Table 1

In a later section (2.4)the student will want some data at intermediate values of load. The values are then used to introduce the idea of physical continuity. One run with intermediate values should be sufficient. A possible selection of loads is 60 to 150 gm at 10 gm intervals.

2.3 Graphing the Experimental Points

The relation between load and position reading must be clearly understood by the student. Note that the horizontal axis is used to designate the load and the vertical axis designates the position reading. Hence, the coordinates of every point which is graphed is an ordered pair for which the first element is the load and the second element is the position reading. The scale on the horizontal axis should go from 0 to 300; the scale on the vertical axis from the reading at no load to that of maximum load.

2.4 Connecting Plotted Points

By connecting poin s the student is inferring physical continuity. He is beginning to develop the physical model, but this does not imply any mathematical continuity. The decision on whether an experiment could yield intermediate points must be based on phenomena being studied. It would be desirable for the teacher to mention a few examples of discontinuous physical situations. For example, there is the maximum height to which a ball bounces. If the height of bounce is related to the corresponding bounce number, physical continuity cannot be inferred. The graph is a set of distinct points which cannot be connected, for height cannot be inferred for parts of bounces. Half bounces cannot be associated with a maximum height.

The sets of ordered pairs in Table 1 are graphed in Figure 1. The individual ordered pairs are connected by line segments for each run of the data.

We will refer to Figure 1 again in Section 2.4, when discussing "the best line".

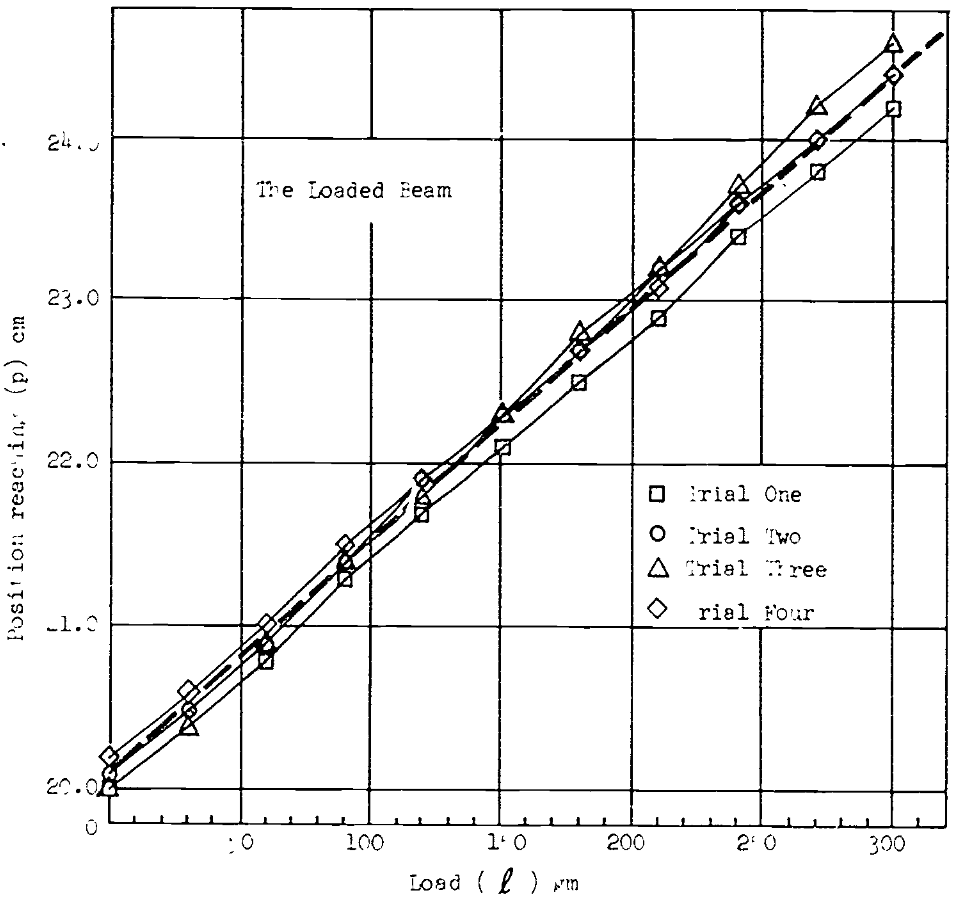


Figure 1

2.5 The Best Line

After plotting the data for three runs and connecting the points of each run, it should be apparent to the student that the graph loses its ability to give information readily. It is now very difficult to obtain information about the beam's behavior from the graph since points of equal load do not coincide. The confusion of points will give the student a strong motive for idealizing the graph and drawing "the best line". This "best line" represents the ideal experiment, and will yield approximate data quickly.

Obtaining the "best straight line" involves making the assumption that the line can be drawn where it should be. Often it will not be too difficult to obtain a line which will satisfy the student. Do not put too much stress on this now. The student will soon have practice, and he should learn quickly.

The dotted line in Figure 1 is the "best line" for the data in Table 1. This line will not be the same as the lines obtained by students in the class. In particular, the slopes and intercepts will differ from group to group because of differences in ruler stiffness and the distance the ruler extends beyond the edge of the desk.

After the physical model has been obtained, the student should then use the ordered pairs determined for the run involving intermediate masses and compare these points to the "best line". If the work has been done carefully, these points should fall close to the "best line" and give the student confidence in constructing and using his physical model.

Exercise 1

1. Referring to your final graph of load-position pairs for the loaded beam, is the horizontal scale drawn along the horizontal axis? No, it is drawn above the horizontal axis. Is the vertical scale drawn along the vertical axis? Yes.

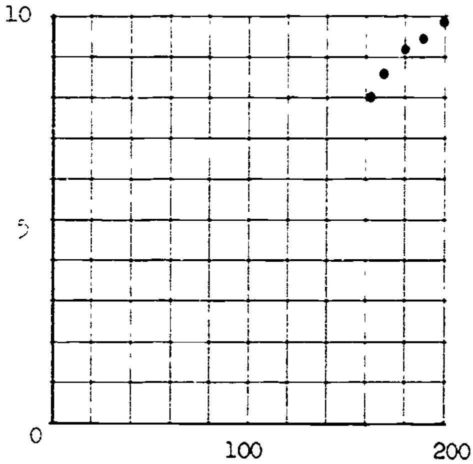
2. Give a good reason why coordinate axes do not always appear on your coordinate paper.

   The graph would not "fill" the paper and hence, the divisions on the axis would have to be shorter.

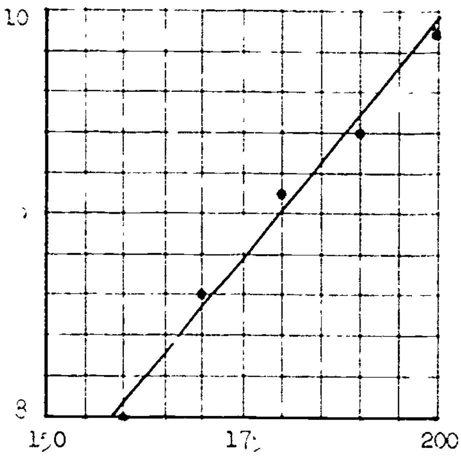
3. On a sheet of coordinate paper, draw horizontal and vertical axes with the origin at the lower left-hand corner. Number the horizontal axis from 0 to 200. Number the vertical axis from 0 to 10. Plot the following set of ordered pairs relating temperature and time:

{(160,8.0), (170,8.6), (180,9.1), (190,9.4), (200,9.9)}

This makes the graph appear on a small portion of the page instead of filling the page.



4. Make a new graph of the points of Exercise 3 in such a way that the graph nearly "fills" the coordinate paper. Label both the horizontal and vertical scales.



5. Draw your "best" straight line through the points plotted in Exercise 4.

The "best" straight line appears on the graph of Problem 4.

Why do some of the points fall off the line?

Probably due to inaccuracy of measurement or of graphing the data.

If the horizontal coordinates are the temperatures of an iron rod in degrees Centigrade, and the vertical coordinates are the corresponding times in minutes, is the drawing of the line justified?

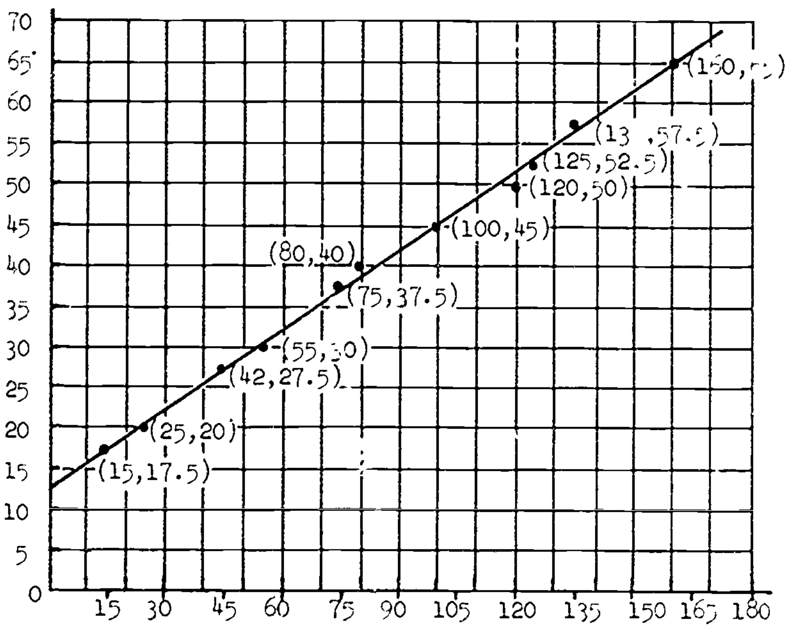
Yes, because intermediate values do exist.

6. Referring to the exercise above, what is the time corresponding to a temperature of 165°C? What is the temperature corresponding to a time of 9.3 minutes?

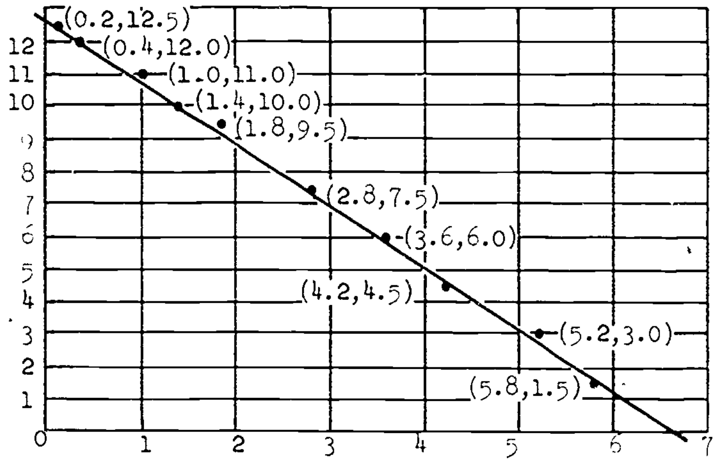
Have students find this on their graph. From the graph in Problem 4, 165° will correspond to 8.3 min; 9.3 min will correspond to 186°.

7. For each of the following, plot the points whose coordinates are given and then draw what you judge to be the best line. Read the y-value of the point at which your line crosses the y axis and compare the results with your classmates.

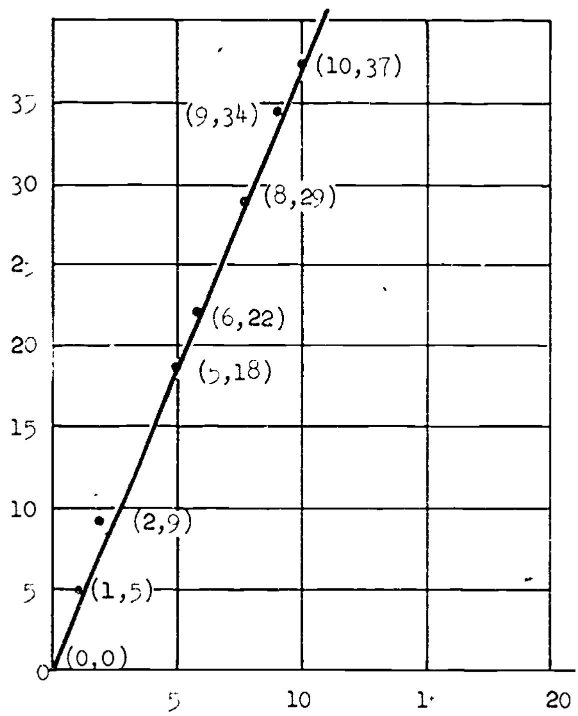
(a) (15,17.5), (25,20.0), (45,27.5), (55,30.0), (75,37.5), (80,40.0), (100,45.0), (120,50.0), (125,52.5), (135,57.5), (160,65.0).



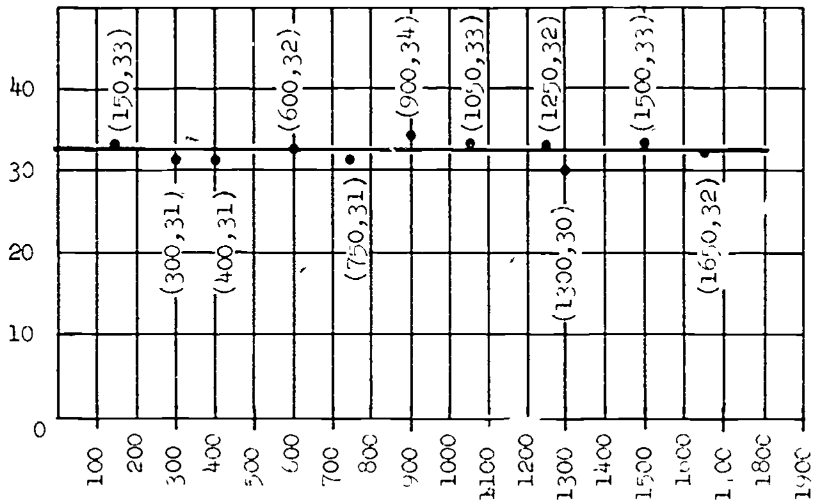
(b) (0.2,12.5), (0.4,12.0), (1.0,11.0), (1.4,10.0), (1.8,9.5), (2.8,7.5), (3.6,6.0), (4.2,4.5), (5.2,3.0), (5.8,1.5).



(c) (0,0), (1,5), (2,9), (5,18), (6,22), (8,29), (9,34), (10,37).



(d) (150,33), (300,31), (450,31), (600,32), (750,31), (900,34), (1050,33), (1250,32), (1300,30), (1500,33), (1650,32).



2.6 Slope

This may be the students' first introduction to slope. Slope may be defined as

$$\frac{\text{vertical change}}{\text{horizontal change}}$$

The vertical change is equivalent to the rise, for this change would be the difference in two values of the range. On the other hand, the horizontal change is equivalent to the run, since this change is the difference in two values of the domain. Therefore, the definition of slope, as given in the text, $\frac{\text{rise}}{\text{run}}$, and the above definition, $\frac{\text{vertical change}}{\text{horizontal change}}$, both describe the same relationship.

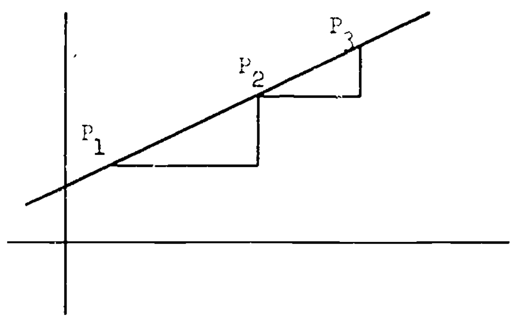


Figure 2

Suppose we have three points, P_1, P_2, P_3, on the same straight line. Triangles can be formed by drawing horizontal and vertical lines between these points as shown in Figure 2. These triangles are similar and, therefore, the ratio between two sides of one triangle is equal to the ratio between corresponding sides of the other triangles.

To find the slope of a particular line, the student must choose two specific points on the line. To obtain two ordered pairs he must then determine their coordinates by comparison with the horizontal and vertical scales. By taking the difference between the second elements of these ordered pairs, he will have the vertical change. The horizontal change may be found by taking the difference between the first elements of the ordered pairs.

If you apply this concept to the "best straight line" for the Loaded Beam graph above, you may choose the points A and B on the line. A is chosen on the load line at 60 and you can determine that the vertical distance to the point is about 20.9. Hence, the horizontal distance is 60 and the vertical distance is 20.9. The second point B has a horizontal distance of 270 and a vertical distance of 24. The difference in the vertical distance is $24 - 20.9 = 3.1$, and the difference in the horizontal distances, taken in the same order, is $270 - 60 = 210$. Hence, the slope is

$$\frac{\text{vertical change}}{\text{horizontal change}} = \frac{3.1}{210} = 0.015$$

The reason that this is so small is that the horizontal distance increases so much more rapidly than the vertical distance.

Exercise 2

1. Which of the following two ordered pairs determine a horizontal line, a vertical line or a line which is neither?

(a)	(3,2), (5,2)	horizontal	(f)	(2,3), (2,2)	vertical
(b)	(0,0), (7,0)	horizontal	(g)	(561,10), (562,11)	neither
(c)	(10,4), (4,10)	neither	(h)	(3,14), (6,28)	neither
(d)	(5,6), (6,7)	neither	(i)	(9,8), (9,1)	vertical
(e)	(2,8), (4,8)	horizontal	(j)	(0,8), (0,5)	vertical

2. For each of the following two ordered pairs, state the rise and the run for the line determined by these points.

(a)	(2,5), (4,8)	rise 3, run 2
(b)	(3,9), (2,1)	rise 8, run 1
(c)	(8.5,7), (9,9)	rise 2, run 0.5
(d)	(20,10), (25,17)	rise 7, run 5
(e)	(5,3), (5,986)	rise 983, run 0
(f)	(763,763), (25,25)	rise 738, run 738
(g)	(8,7), (2,5)	rise 2, run 6
(h)	(8,10), (0,10)	rise 0, run 8
(i)	(3.7, 12.6), (5.2,13.1)	rise .5, run 1.5
(j)	$(\frac{3}{4},\frac{5}{6})$, $(\frac{5}{4},\frac{11}{6})$	rise 1, run $\frac{1}{2}$

3. Determine the slope of the line connecting the points in each part of Problem 2.

(a)	$\frac{3}{2}$	(f)	1
(b)	8	(g)	$\frac{1}{3}$
(c)	4	(h)	0
(d)	$\frac{7}{5}$	(i)	$\frac{1}{3}$
(e)	has no slope	(j)	2

2.7 Equation of a Straight Line -- Slope-Intercept Form

It is now necessary to make a more detailed analysis of the linear equation and its graphical representation.

The y-intercept, as stated in the student's text, is the point where the line crosses the y-axis. In this section, the student will read this point from the graph to determine that the coordinates of the point are (0,b). To find the equation of this line, he must be able to see that the slope of the line is always the same along the total length of the line. He can then write an expression for this slope using the y-intercept (0,b) and an arbitrary point (ℓ,p) which is anywhere on the line. Equating this expression with the numerical value of the slope, he then geta an equation of the line in terms of ℓ and p,

$$\frac{p - b}{\ell - 0} = m .$$

Application of algebraic manipulation then gives the equation

$$p = m\ell + b ,$$

which is the slope-intercept form of the equation of a straight line.

From the graph of the loaded beam, the value of b can be read as 20.1. In the previous section, the slope, m, of the line was found to be 0.015. Hence, the equation of the "best straight line" is

$$p = 0.015\ell + 20.1 .$$

All students should find the equation of their "best straight line".

The example given in the student text, referring to Figure 12, deter mines the slope of the line as $m = \frac{5}{4}$. From the graph he reads the coordinates of the y-intercept as (0,8). Now the slope has been defined as the $\frac{\text{rise}}{\text{run}}$, or $\frac{\text{vertical change}}{\text{horizontal change}}$. In the above problem, the arbitrary aecond point has coordinates (ℓ, p). Using the first point as the y-intercept, (0,8), the vertical and horizontal changes would give a slope of

$$\frac{p - 8}{\ell - 0} = \frac{5}{4} \text{ or } \frac{p - 8}{\ell} = \frac{5}{4} .$$

Solving this equation, we multiply by ℓ .

$$\ell\left(\frac{p - 8}{\ell}\right) = \ell\left(\frac{5}{4}\right)$$

Since $\frac{\ell}{\ell} = 1$, $p - 8 = \frac{5}{4}\ell$.

Adding 8, we obtain $p - 8 + 8 = \frac{5\ell}{4} + 8$,

which gives the equation

$$p = \frac{5\ell}{4} + 8 \text{ or } p = \frac{5}{4}\ell + 8.$$

Students may use this process for finding the equation of their "best straight line".

Exercise 3

1. Tabulate the coordinates of the points P, Q and R, shown in the accompanying graph. Calculate the slope of line ℓ_1, using the points P and Q. Do the same for points P and R, and again for points Q and R.

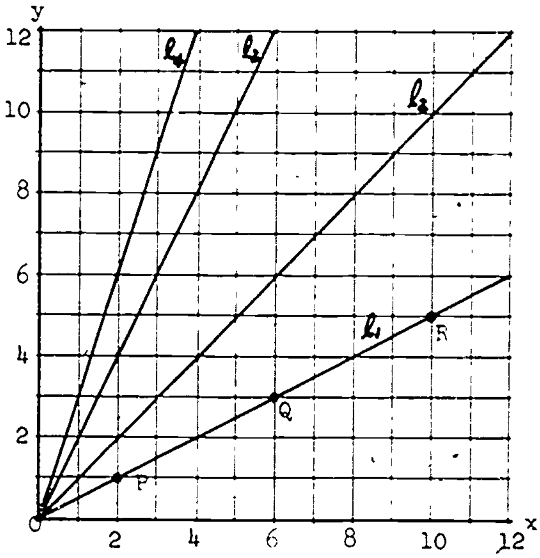


Figure 14

P(2,1), Q(6,3), R(10,5)

$$m_{\overleftrightarrow{PQ}} = \frac{3 - 1}{6 - 2} = \frac{2}{4} = \frac{1}{2}$$

$$m_{\overleftrightarrow{PR}} = \frac{5 - 1}{10 - 2} = \frac{4}{8} = \frac{1}{2}$$

$$m_{\overleftrightarrow{QR}} = \frac{5 - 3}{10 - 6} = \frac{2}{4} = \frac{1}{2}$$

2. Referring to Figure 14, what are the slopes and vertical axis intercepts of lines ℓ_2, ℓ_3 and ℓ_4 ?

slope ℓ_2 : $\frac{6 - 0}{6 - 0} = 1$

slope ℓ_3 : $\frac{12 - 0}{6 - 0} = 2$

slope ℓ_4 : $\frac{9 - 0}{3 - 0} = 3$

The intercept for all four lines is 0 .

3. Find the slope and intercept of lines ℓ_5 and ℓ_6 (Figure 15).

ℓ_5 : slope $\frac{1}{3}$ intercept 0

ℓ_6 : slope $\frac{1}{3}$ intercept 3

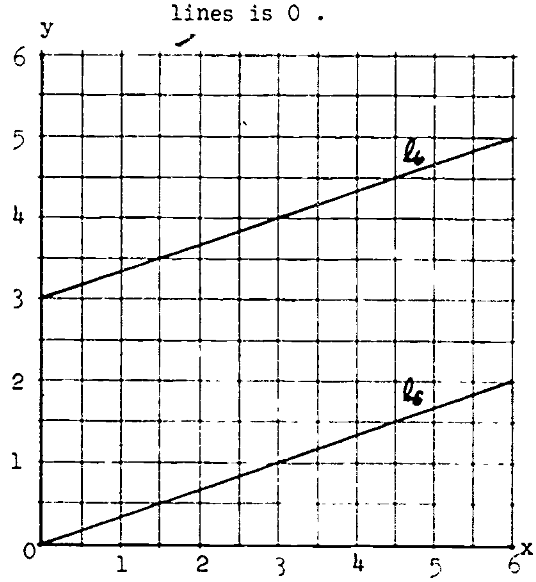


Figure 15

4. What do you need to know about a line to distinguish it from any other line?

   Its slope <u>and</u> y-intercept.

5. Write the equations for the lines ℓ_1, ℓ_2, ℓ_3, ℓ_4, ℓ_5 and ℓ_6, shown in Figures 14 and 15.

   $\ell_1 : \ y = \frac{1}{2}x$ $\qquad$ $\ell_4 : \ y = 3x$

   $\ell_2 : \ y = x$ $\qquad$ $\ell_5 : \ y = \frac{1}{3}x$

   $\ell_3 : \ y = 2x$ $\qquad$ $\ell_6 : \ y = \frac{1}{3}x + 3$

6. In the Loaded Beam Experiment, what is the significance of the position axis intercept that you obtained?

   It was the position on the meter stick from which the amount of deflection was measured.

   Would a different intercept have given you a different slope?

   No. The intercept is determined by the position of the pointer and the meter stick, while slope is determined by the amount the beam bends.

2.8 Graphing Linear Equations

The student must become very familiar with the slope in many of its aspects. At a later time he will learn that the slope not only tells the "steepness" of a line, but whether it rises or falls as we proceed from left to right.

The horizontal line, which has a slope of zero, is not difficult to understand. The student usually recognizes that $\frac{0}{a} = 0$, and on a horizontal line there is no vertical change or rise.

However, the vertical line is more difficult for the student. Since there is no horizontal change, the slope would take the form $\frac{a}{0}$. But there is no definition for the division $\frac{a}{0}$, and hence the slope of a vertical line is undefined.

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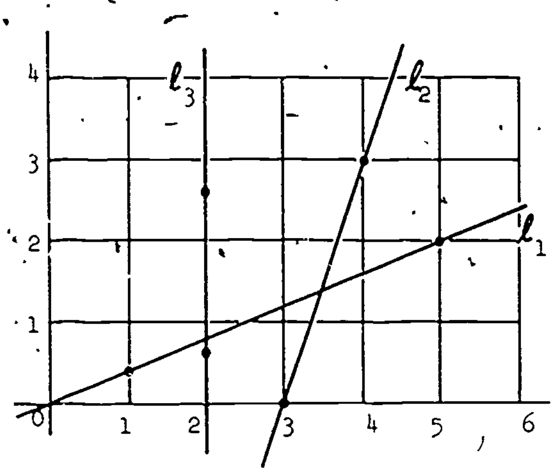
Exercise 4

1. Calculate the slopes of lines ℓ_1, ℓ_2 and ℓ_3 in the accompanying figure, using in each case the two points indicated on the lines.

slope ℓ_1 : $\frac{2 - 0.4}{5 - 1} = 0.4$

slope ℓ_2 : $\frac{3 - 0}{4 - 3} = 1$

slope ℓ_3 : $\frac{2.6 - 0.6}{2 - 2}$ is undefined



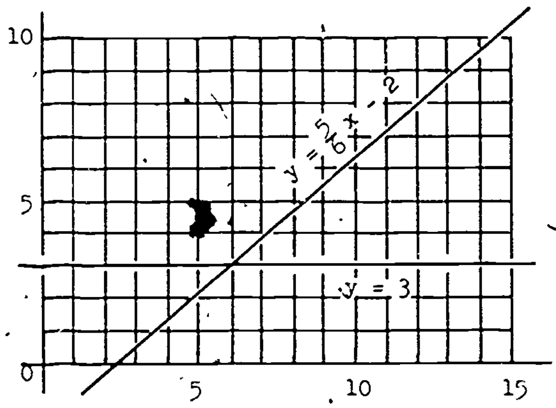
2. What is the slope of a horizontal axis? 0.

What is the slope of a vertical axis? Undefined.

3. With reference to a set of coordinate axes, select the point (6,3) and through this point

(a) draw the line whose slope is $\frac{5}{6}$. What is the equation of this line?

(b) draw the line through (6,3) which has a slope of zero. What is an equation of this line?



4. Draw the following lines:

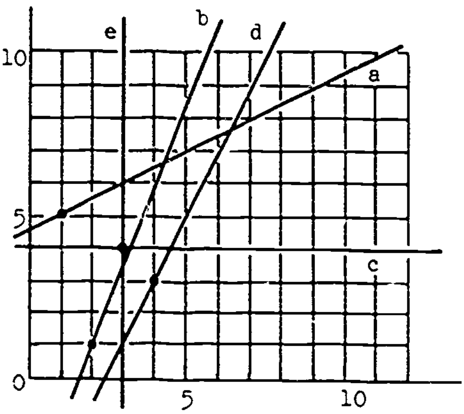
(a) through (1,5) with slope $\frac{1}{2}$.

(b) through (2,1) with slope $\frac{5}{2}$.

(c) through (3,4) with slope 0.

(d) through (4,3) with slope 2.

(e) through (3,4) with slope undefined. (What type of line has no defined slope?)



5. Consider the line containing the points (2,3) and (9,5). Which of the following points is on this line?

(Hint: First determine the slope of the line containing the points (2,3) and (9,5))

$$m = \frac{5-3}{9-2} = \frac{2}{7}$$

An easy method for checking the various points is to compare slopes of lines through (2,3) and the given point with $\frac{2}{7}$.

(a) (30,11) $m = \frac{11-3}{30-2} = \frac{8}{28} = \frac{2}{7}$ On the line.

(b) (7,4) $m = \frac{4-3}{7-2} = \frac{1}{5}$ Not on the line.

(c) (22,9) $m = \frac{9-3}{22-2} = \frac{6}{20}$ Not on the line.

(d) (23,9) $m = \frac{9-3}{23-2} = \frac{6}{21} = \frac{2}{7}$ On the line.

(e) (19,58) $m = \frac{58-3}{19-2} = \frac{55}{17}$ Not on the line.

(f) (58,19) $m = \frac{19-3}{58-2} = \frac{16}{56} = \frac{2}{7}$ On the line.

6. Write an equation of each of the following lines.

(a) The slope is $\frac{2}{3}$ and the y-intercept number is 2. (The y-intercept number is the vertical coordinate of the point at which the line crosses the vertical axis. In this case the coordinates of the intercept are (0,2).) $y = \frac{2}{3}x + 2$

(b) The slope is $\frac{3}{4}$ and the y-intercept number is 0. $y = \frac{3}{4}x$

(c) The slope is $\frac{5}{3}$ and the y-intercept number is $\frac{4}{3}$. $y = \frac{5}{3}x + \frac{4}{3}$

(d) The slope is 37 and the y-intercept number is 5. $y = 37x + 5$

7. What is the slope of the line containing the points (0,0) and (3,4)?

$$m = \frac{4 - 0}{3 - 0} = \frac{4}{3}$$

What is the y-intercept?

y-intercept : 0

Write the equation of the line.

$$y = \frac{4}{3}x$$

8. Verify that the slope of the line which contains the points (0,5) and (8,13) is 1.

$$m = \frac{13 - 5}{8 - 0} = \frac{8}{8} = 1.$$

If (x,y) is a point on this same line, the slope could be written as

$$m = \frac{y - 5}{x - 0} \quad \text{or} \quad m = \frac{y - 13}{x - 8}.$$

Show that both expressions for the slope give the same equation for the line.

$$\frac{y - 5}{x - 0} = 1 \qquad\qquad \frac{y - 13}{x - 8} = 1$$

$$\begin{aligned} y - 5 &= 1 \cdot (x - 0) \\ y - 5 + 5 &= 1 \cdot x + 5 \\ y &= x + 5 \end{aligned} \qquad\qquad \begin{aligned} y - 13 &= 1 \cdot (x - 8) \\ y - 13 + 13 &= 1 \cdot x - 8 + 13 \\ y &= x + 13 - 8 \\ y &= x + 5 \end{aligned}$$

9. Write the equations of the lines through the following pairs of points. Use the method of Problem 8.

(a) (0,3) and (5,12)	$y = \frac{9}{5}x + 3$
(b) (5,8) and (0,4)	$y = \frac{4}{5}x + 4$
(c) (0,2) and (3,7)	$y = \frac{5}{3}x + 2$
(d) (5,8) and (0,6)	$y = \frac{2}{5}x + 6$
(e) (3,0) and (6,3)	$y = x - 3$
(f) (3,3) and (5,3)	$y = 3$
(g) (3,3) and (3,5)	Vertical line such that the x-coordinate is always 3 x = 3 is the equation of this line.
(h) (4,2) and (3,1)	$y = x - 2$

2.9 Relations and Functions

In discussing ordered pairs, it is possible to consider seating arrangements in the classroom. The seat position a student holds in the classroom may be expressed in terms of row and seat number. Row 2 seat 5 may be the seat occupied by John. This may be expressed as an ordered pair (2,5). Mary may be assigned to row 3 seat 6, expressed as (3,6), and Susan to row 5 seat 2, (5,2). Unless the importance of order is stressed, the seats assigned to John, (2,5), and Susan, (5,2), may be confused. If John were listed by row and seat number, and Susan by seat and row number, the ordered pairs would appear to be the same, (2,5), and again there would be confusion. It is therefore very important that order be stressed. In fact, it is a necessity if consistent understanding and agreement is to be reached.

A relation may be defined as a set of ordered pairs. A set, M, of ordered pairs defines a relation R as follows: xRy, (x is related to y), is true if there is an element of M having x as its first member and y as its second member; otherwise, xRy is false. The set of first elements of the ordered pairs in M is the domain, and the set of second elements is the range.

It is now necessary to expend student understanding of domain and range in a specific manner. The concept of a function can now be discussed. Its definition is clear and specific:

> A set of ordered pairs defines a function if and only if no two ordered pairs of the set have the same first element.

To each number of the domain the function assigns one and only one number from the range.

In the discussion about functions, it is important to emphasize the following points:

(1) To each number of the domain of definition, the function assigns one and only one number from the range. However, the same number of the range can be assigned to many different elements of the domain.

(2) The essential idea of function is found in the actual association from numbers in the domain to numbers in the range, and not in the particular way in which the association happens to be described.

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(3) Always speak of the assoc ion as being from the domain to the range. This helps fix the correct idea that we are dealing with an ordered pairing of numbers in which the number from the domain is mentioned first and the assigned number from the range is mentioned second.

(4) Not all functions can be represented by algebraic expressions.

Although the above points are not absolutely vital as far as elementary work with functions is concerned, they become of central importance at a later time. Also, many of the difficulties which students have with the idea of function can be traced to confusion in these matters. Therefore, it is important that the student understands these points from his very first contact with the function concept.

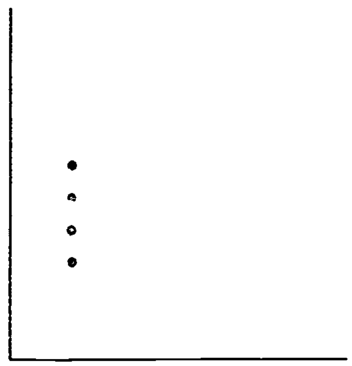
Exercise 5

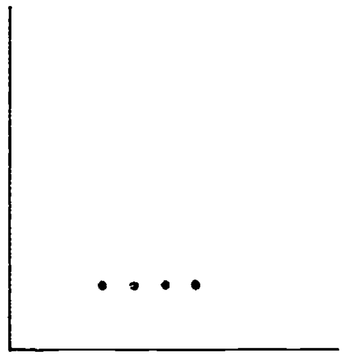
In Problems 1 through 5

(a) Graph the ordered pairs given below, state the domain and the range, and tell if the relation is a function.

(b) In each case form a new relation by interchanging the first and second elements of the ordered pairs. Graph this relation, state the domain and range, and tell if it is a function

1. $M = \{(2,3), (2,4), (2,5), (2,6)\}$

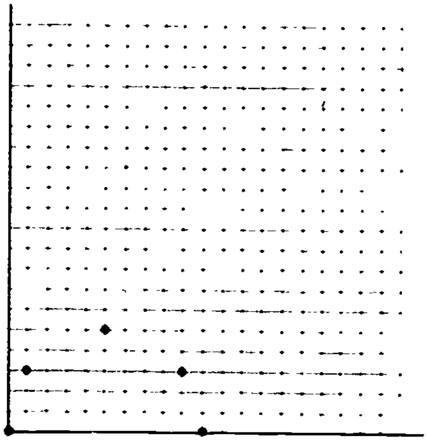


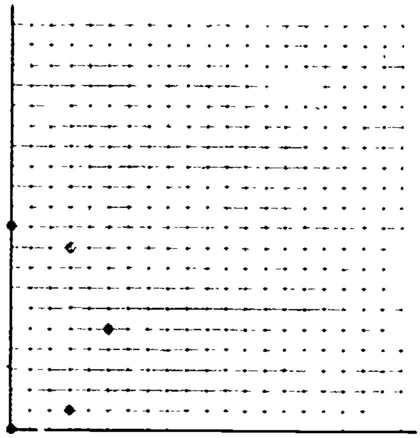


domain {2}, range {3,4,5,6}, relation is not a function

new domain {3,4,5,6}, range {2}, relation is a function

2. N = {(0,0), (1,3), (5,5), (9,3), (10,0)}

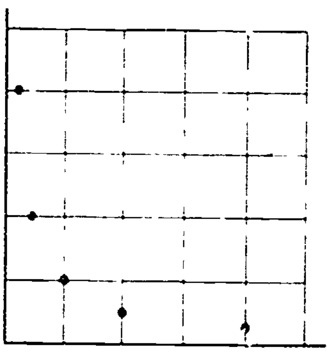


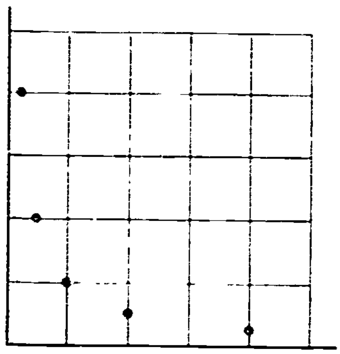


domain {0,1,5,9,10}
range {0,3,5}
relation is a function

new domain {0,3,5}
new range {0,1,5,9,10}
new relation is not a function

3. P = {($\frac{1}{4}$, [illegible]), ($\frac{1}{2}$,2), (1,1), (2,$\frac{1}{2}$), ([illegible],$\frac{1}{4}$)}

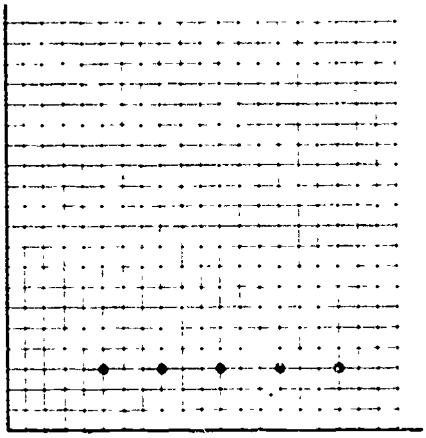


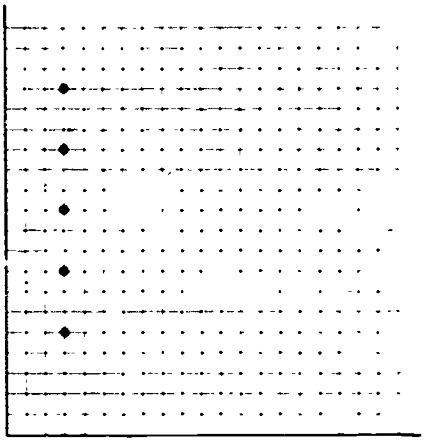


domain {$\frac{1}{4}$, $\frac{1}{2}$, [illegible], [illegible], [illegible]}
range {$\frac{1}{4}$, $\frac{1}{2}$, [illegible], 2, [illegible]}
relation is a function

new domain {$\frac{1}{4}$, $\frac{1}{2}$, 1, 2, [illegible]}
new range {$\frac{1}{4}$, $\frac{1}{2}$, [illegible], 2, [illegible]}
new relation is a function

4. Q = {(5,3), (8,3), (11,3), (14,3), (17,3)}





domain {5,8,11,14,17}

range {3}

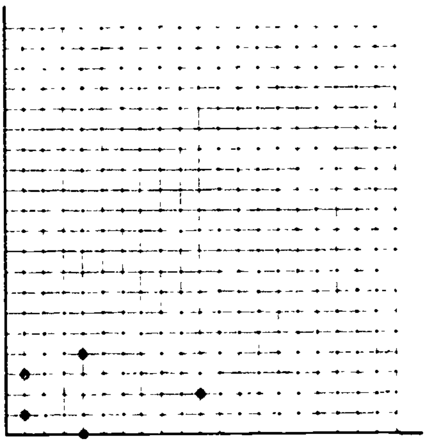
relation is a function

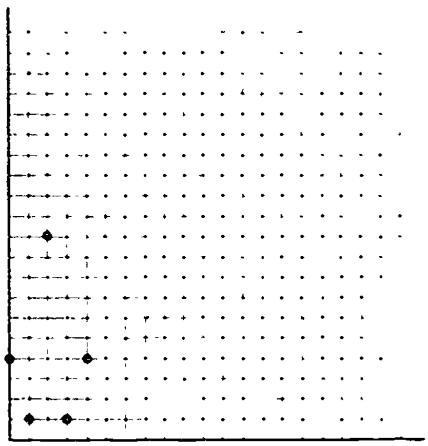
new domain {3}

new range {5,8,11,14,17}

new relation is not a function

5. T = {(10,2), (1,1), (1,3), (4,0), (4,4)}





domain {1,4,10}

range {0,1,2,3,4}

relation is not a function

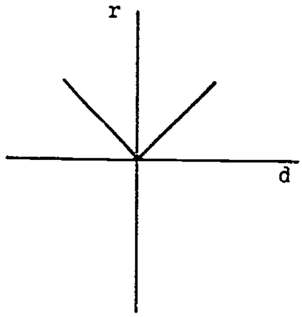
new domain {0,1,2,3,4}

new range {1,4,10}

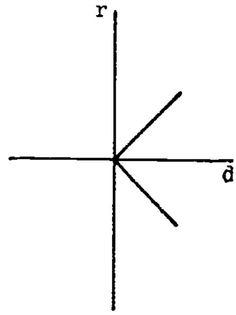
new relation is a function

6. Which of the graphs of the relations shown below are graphs of a function?

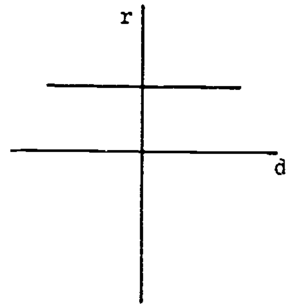
(a) function



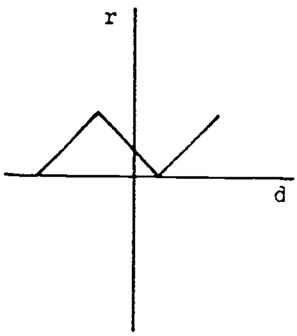
(b) not a function



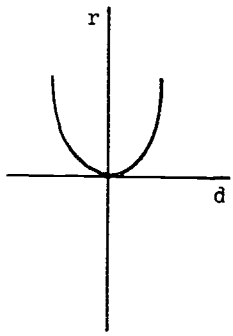
(c) function



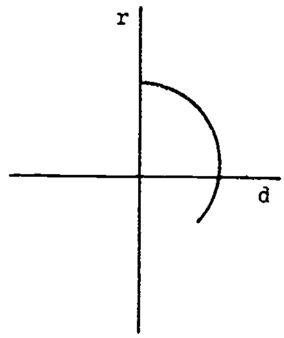
(d) function



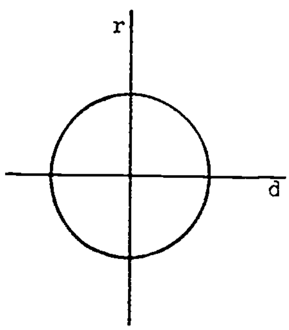
(e) function



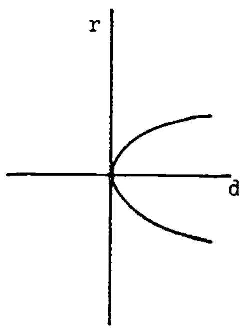
(f) not a function



(g) not a function



(h) not a function



7. In the Loaded Beam Experiment the data in the table forms a relation.

   (a) What are the domain and the range of this relation?

   domain {0,30,60,90, ..., 300}

   range {20,20.4,20.8, ..., 24.6}

   (b) Is this relation a function?

   Yes.

8. Does the "best straight line" describe a function?

   Yes.

9. Are the domain and range of the "best straight line" relation the same as the domain and range of the "data relation"? Explain.

   No. The "best straight line relation" includes all of the values of d from 30 to 300, inclusive, and from 20 to 24.6 inclusive while the "data relation" is only a discrete sampling.

10. Are the domain and range of the equation found to represent the "best straight line" the same as the domain and range of the best straight line relation?

    Yes.

2.10 The Falling Sphere

The equipment needed by each group performing this experiment is as follows:

1 glass cylinder or jar at least 8 inches high
1 steel ball bearing, about $\frac{1}{8}$-inch diameter
1 small magnet
1 12-inch ruler, also calibrated in centimeters
1 metronome
1 bottle Karo syrup (white)
4 paper strips, about 1" × 10"
1 roll cellophane tape

This experiment is an investigation of terminal velocity. A small steel ball bearing dropped into a cylinder of Karo syrup will reach its terminal velocity very near the top of the cylinder.

Affix a thin paper tape to a cylinder or jar. Fill the cylinder with a fluid such as white Karo syrup Drop a small steel ball bearing into the fluid close enough to the tape to make fairly accurate marks on the tape at regular time intervals. A metronome is a convenient device for marking the equal time intervals, which should be about two seconds. After the ball bearing has been dropped into the fluid, put a pencil mark on the tape indicating the position of the ball bearing at each click of the metronome. The students will find this easy to do after a little practice. It seems to be a matter of getting into the proper rhythm in making the marks. The magnet is used to retrieve the ball bearing after it has come to rest on the bottom of the cylinder. The tape for each trial must be saved for use in the next section of the text.

Should a metronome be difficult to obtain, it may be possible to borrow one for a short time. Make a tape recording of the metronome sounding at the correct intervals. This tape can be played loudly enough for the use of the entire class.

Since, in this experiment, the students are considering terminal velocity, they might consider the differences that might occur if the same bearing was to be dropped in other fluids. It would be advantageous, as a teacher demonstration, to use water, glycerine, and a light oil as fluids in the experiment. A few trials with each fluid should lead the students to reach conclusions as to the effect of the fluid on the velocity.

Again, as a demonstration, cylinders or jars with different size openings should be used with the same fluids as above. The students can then reach a conclusion concerning the effect of the type of container on the terminal velocity.

The student experiment should be performed with a single size of container, and a single fluid, preferably white Karo syrup. Four trials will be needed to provide the necessary data.

2.11 The Graph and the Equation

The tapes obtained from the trials in the experiment in Section 3.9 must now be measured. Each tape in turn should be fastened to centimeter rule. The zero mark on the tape should coincide with an integral value of the ruler, e.g., 1,2,3, ... The zero mark on the ruler may be used if it is sufficiently far from the end of the ruler to allow accurate placing of the tape. The integral value of the ruler is used to facilitate reading the measure of the distance between the zero mark and each succeeding mark on the tape. Since these distances are to be plotted on the coordinate plane as dependent on the time interval, accuracy is important. The various errors inherent in the results obtained by student experiment should provide a "braid" rather than a single line when the graph is completed. This will probably occur despite a plea for accuracy.

From the "braid" obtained as a result of graphing the four trials of the experiment, a "best" straight line can be drawn. This line is an idealization of the graph of the physical data. But this idealization does not represent any possible physical situation. It is purely an abstraction.

The slope can now be determined by the student. This slope will have a special significance. Note that the vertical distance is a measure of the fall of the ball bearing, and the horizontal distance is a measure of the corresponding measure of time. Therefore, the slope becomes

$$\frac{\text{measure of change in distance}}{\text{measure of change in time}}$$

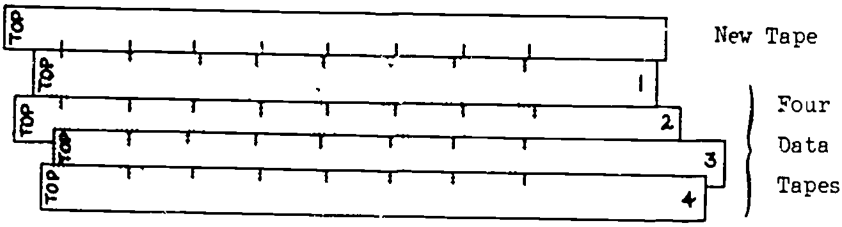
or, distance per unit of time, which is velocity.

Since the data graphed determines a straight line, the velocity of the falling ball bearing is constant. This substantiates the statement that terminal velocity had been reached by the time of the initial reading.

Exercise 6

The purpose of the first two problems in this exercise is to give alternate ways to find the best straight line from the data already collected. The actual answer obtained will depend upon the data collected by the students. The problems are repeated here for the convenience of the teacher.

1. Reproduce the "best straight line" you have drawn to represent the data of this experiment on a clean sheet of coordinate paper. Take the four pieces of paper tape used to mark the position of the ball and arrange them so that the zero marks are in line. On a clean fifth tape make a mark to indicate a "zero" position and align this mark with the other zero marks. The other marks on your tapes will not be "in line", but should tend to center in groups about a number of imaginary vertical lines. Make a mark on the clean tape to indicate your "guess" as to the position which best represents each vertical set of marks. Using the fifth tape as if it were a new trial, mark your measurements in the usual way, enter the data in your table, and graph the ordered pairs. Do

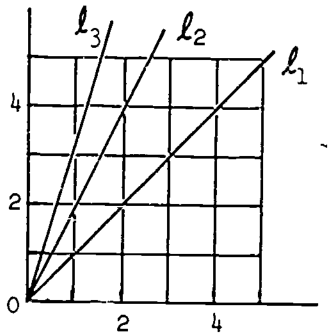


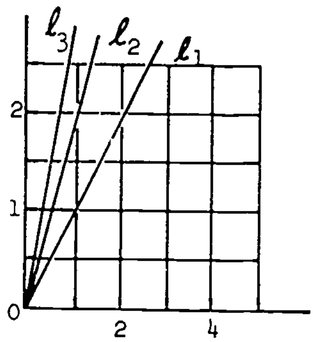
these points come closer to forming a straight line than any of your four trial runs? How does this line compare with the "guess" you made from the "braid" arrangement?

2. From the data of your four trials, find the average distance traveled by the ball in each time interval. Make a new column in your table, "Average Distance (mm)", and now plot average distance versus time on the same sheet of coordinate paper used for Exercise 1. How close do these points come to forming a straight line? You now have three lines on this sheet of coordinate paper. The first is the "best straight line" from your original data, the second is the line obtained in Exercise 1, and the third line is the one obtained by the process of averaging. How do these three lines compare?

3. Draw a graph using a scale of 1 second for each horizontal division and 1 millimeter for each vertical division. Draw a line which passes through the origin and has a slope of 1 mm/sec, 2 mm/sec, and 3 mm/sec. Label these lines

4. Repeat Exercise 3 with a horizontal scale of 1 second per division, but with a vertical scale of 0.5 millimeter per division.





Are the two slopes the same? Yes, since slope is defined by

$$m = \frac{\text{vertical change}}{\text{horizontal change}} .$$

For example, the slope of ℓ_1 in both cases is $m = \frac{1}{1} = 1$.

2.12 The Point-Slope Form

There are several forms for the equation of a straight line. In this section, the point-slope form is developed. Then this form is reduced to the slope-intercept form.

The ordered pair for the point at which the graph of the line crosses the horizontal axis would have the form (a,0). If w choose any arbitrary point on the line having coordinates (x,y). The slope, m, which has been described as $\frac{\text{vertical change}}{\text{horizontal change}}$ will be written as

$$\frac{y - 0}{x - a} = m$$

or

$$\frac{y}{x - a} = m$$

If the x-intercept is not known but some point (a,b) is known, then the equation becomes

$$\frac{y - b}{x - a} = m \ ,$$

which is the point-slope form of the linear equation.

Exercise 7

1. Write the equations of the lines ℓ_1, ℓ_2 and ℓ_3, using the two points indicated in the following graph.

$\ell_1 : m = \frac{8 - 0}{17 - 11} = \frac{4}{3}$

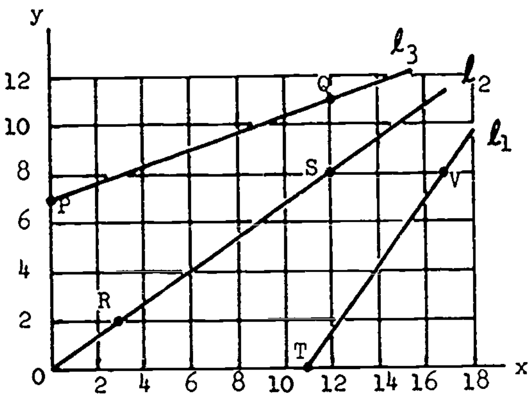
$\frac{y}{x - 11} = \frac{4}{3}$

$\ell_2 : m = \frac{8 - 2}{12 - 3} = \frac{6}{9} = \frac{2}{3}$

$\frac{y - 2}{x - 3} = \frac{2}{3}$

$\ell_3 : m = \frac{11 - 7}{12 - 0} = \frac{4}{12} = \frac{1}{3}$

$\frac{y - 7}{x} = \frac{1}{3}$



The above answers are quoted in the point-slope form of the equation. However, the teacher should be careful to recognize that equivalent forms of these equations are also correct.

2. Write the equation of the lines ℓ_4 and ℓ_5, using the points indicated in the following graph.

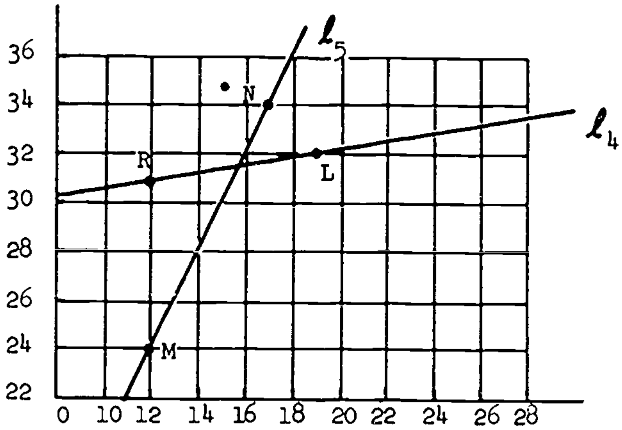
$$\ell_4 : m = \frac{32 - 31}{19 - 12} = \frac{1}{7}$$

$$\frac{y - 31}{x - 12} = \frac{1}{7}$$

$$\ell_5 : m = \frac{34 - 24}{17 - 12} = \frac{10}{5} = 2$$

$$\frac{y - 24}{x - 12} = 2$$

(This simplifies to $y = 2x$.)



3. Refer to your load-position graph obtained in the Loaded Beam Experiment. Using a point not on the vertical axis together with the slope, find the equation to represent the best straight line. Show that this is equivalent to the equation obtained, using the slope-intercept form. (The answers here will vary from student to student. If the students have too much difficulty with algebraic operations, you may wish to omit this problem.)

4. State the slope of the graph of each of the following equations. Give the coordinates of three points on the graph of each.

(a) $\frac{y - 6}{x + 4} = 3$

$y = 3x + 18$ slope : 3, (0,18), (1,21), (2,24), etc.

(b) $\frac{y - 2}{x - 3} = \frac{3}{5}$

$y = \frac{3}{5}x + \frac{1}{5}$ slope : $\frac{3}{5}$, $(0,\frac{1}{5})$, (3,2), (8,5), etc.

(c) $\frac{y - 10}{x - 10} = 1$

$y = x$ slope : 1, (0,0), (1,1), (2,2), etc.

(d) $2(\frac{y - 5}{x - 15}) = \frac{2}{3}$

$y = \frac{x}{3}$ slope : $\frac{1}{3}$, (0,0), (3,1), (6,2), etc.

(e) $5y = 2x + 15$

$y = \frac{2}{5}x + 3$ slope : $\frac{2}{5}$, (0,3). (5,5), (10,7), etc.

(f) $3x + 5 = y$

$y = 3x + 5$ slope : 3, (0,5), (1,8), (2,11), etc.

(g) $y + 4 = \frac{5}{2}(x - 2)$

$y = \frac{5}{2}x - 9$ slope : $\frac{5}{2}$, (4,1), (6,6), (8,11), etc.

(h) $\frac{x - 2}{y - 5} = \frac{9}{10}$

$y = \frac{10}{9}x + \frac{25}{9}$ slope : $\frac{10}{9}$, $(0,\frac{25}{9})$, $(1,\frac{35}{9})$, $(2,\frac{45}{9})$, etc.

Sample Test Items

I. TRUE-FALSE

1. When we graph experimental data, the scales we use on the graph paper must intersect at the origin.
2. It is best to have a graph "fill" the graph paper as much as possible.
3. Scientists and mathematicians are usually satisfied with the results of a single trial when performing an experiment.

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4. Different groups performing the same experiment will always have the same "best straight line".

5. The slope of a given line depends on the particular pair of points used to determine the line and may be different for different points.

6. Slope is positive for a line which rises to the right.

7. Slope is undefined for a horizontal line.

8. The equation $2y = 6x + 3$ is in the slope-intercept form.

9. We can draw a line if we know its slope and its vertical-axis intercept.

II. MATCHING

1. The physical model of an ideal experiment
2. Axis of the range
3. The ratio of vertical change to horizontal change for a straight line
4. Axis of the domain
5. An equation in point-slope form
6. A relation in which each element of the domain is associated with exactly one element of the range
7. An equation in slope-intercept form
8. The set of first elements within a set of ordered pairs that forms a relation.
9. The equation of a line that passes through the origin
10. The set of second elements within a set of ordered pairs that forms a relation

a. Best straight line
b. Domain
c. Function
d. Horizontal axis
e. Range
f. Slope
g. Vertical axis
h. $y = 3x$
i. $y = 3x + 7$
j. $\frac{y - 4}{x - 4} - 3$

III. COMPLETION

1. The ratio of vertical change to horizontal change is called the __(a)__ of the line. Its symbol is __(b)__ .

2. The ______ is the point where the graph meets the vertical axis.

3. In graphing a function, the elements of the ______ are generally represented on the vertical axis.

4. The slope of a vertical line is ______.

5. The slope of a line through (0,0) and (3,4) is ______.

6. The y-intercept of a line through (0,0) and (3,4) is ______.

7. The equation of a line through (0,0) and (3,4) is ______.

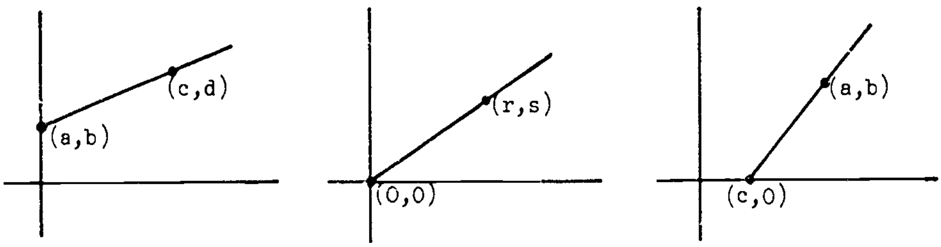
8. Lines having zero slope are ______.

9. The slope of the "best" straight line in the Falling Sphere Experiment was actually a measure of the ______ of the falling sphere.

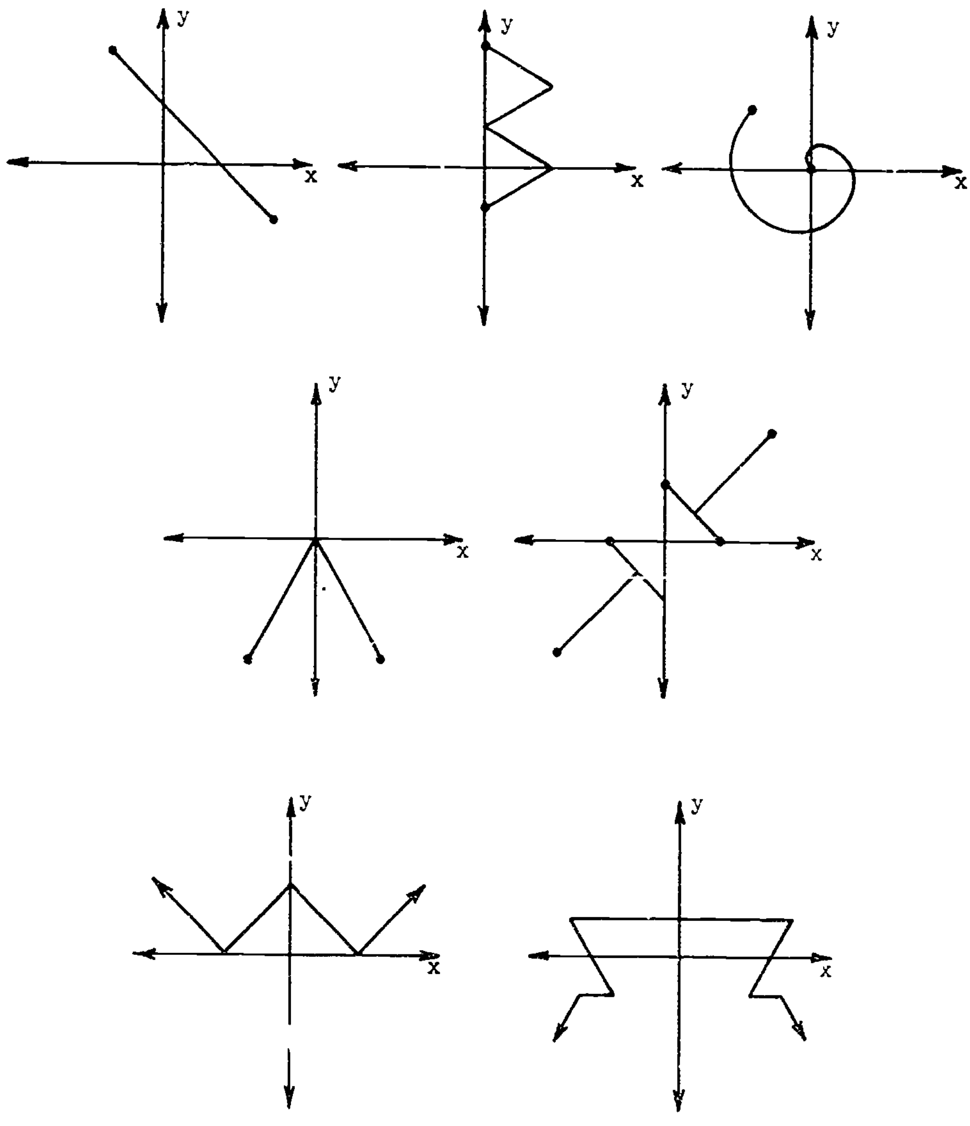
IV. PROBLEMS

1. Using the coordinates shown, find the slope m for each graph and write the equation for each.

(a) (b) (c)



2. Which of the graphs of the relations shown below are graphs of a function?



3. Given the points (3,8) and (14,10), write the equation of the line which passes through these points.

4. Suppose the following data had been obtained in a Loaded Beam Experiment similar to the one we performed.

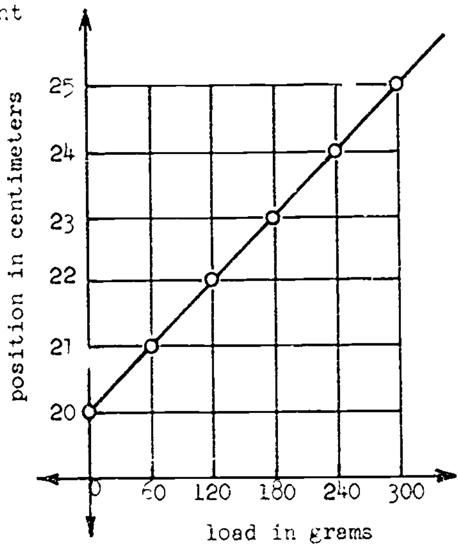
THE LOADED BEAM EXPERIMENT			
Load (grams)	Trial 1 Position (centimeters)	Trial 2 Position (centimeters)	Trial 3 Position (centimeters)
0	20.0	20.1	20.1
60	20.8	20.9	20.9
120	21.7	21.9	21.8
180	22.5	22.7	22.8
240	23.4	23.6	23.7
300	24.2	24.4	24.6

(a) Graph the data from the various trials, carefully selecting the scales.

(b) Determine a "best straight line".

5. Suppose the line at the right had been your best line for the Loaded Beam Experiment. Using the process of interpolation, what would be the position for a load of 150 grams?

____



6. Given the equation $y = 2x + 4$, complete the following table.

x	y
0	
3	
1	
$\frac{1}{2}$	
4	

Graph this set of ordered pairs.

7. Given the equation $y - 6 = 3(x + 4)$.

   Determine the slope and the y-intercept of the graph of the line.

8. Given (0,4) as a point on a line. Graph this line if $m = \frac{1}{3}$. Write the equation of the line.

9. What is the equation of a line through (0,5) and (2,7) ?

Solutions for Sample Test Items

I. TRUE-FALSE

1. False
2. True
3. False
4. False
5. False
6. True
7. False
8. False
9. True

II. MATCHING

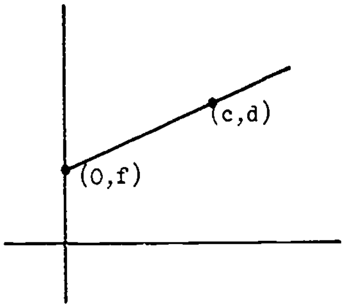
1. (a) Best straight line
2. (g) Vertical axis
3. (f) Slope
4. (d) Horizontal axis
5. (j) $\frac{y - 4}{x - 4} = 3$
6. (c) Function
7. (i) $y = 3x + 7$
8. (b) Domain
9. (h) $y = 3x$
10. (e) Range

III. COMPLETION

1. (a) Slope
   (b) m
2. y-intercept
3. range
4. undefined
5. $\frac{4}{3}$
6. origin
7. $y = \frac{4}{3}x$
8. horizontal
9. velocity

IV. PROBLEMS

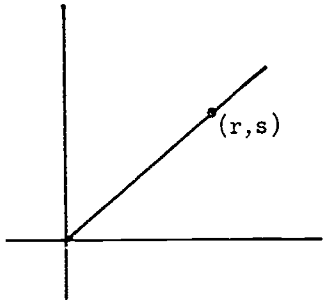
1. (a)



slope = $\frac{d - b}{c}$

equation: $y = (\frac{d - b}{c})x + b$

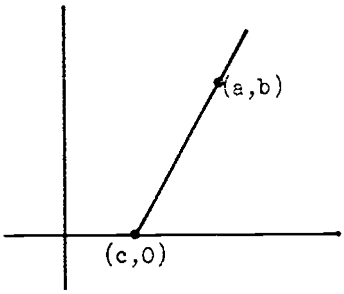
(b)



slope = $\frac{s}{r}$

equation: $y = \frac{s}{r}x$

(c)



slope = $\frac{b}{a - c}$

equation: $\frac{y}{x - c} = \frac{b}{a - c}$ (Any equivalent form of this equation should be accepted as a correct answer.)

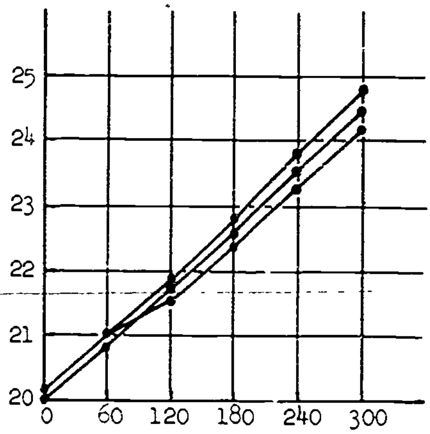
2. (a) function
   (b) not a function
   (c) not a function
   (d) function
   (e) not a function
   (f) function
   (g) not a function

3. $(3,8), (14,10)$

$$m = \frac{10 - 8}{14 - 3} = \frac{2}{11}$$

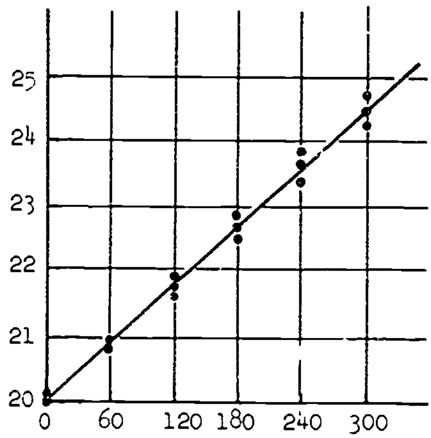
Equation: $\left.\begin{aligned} \frac{y - 8}{x - 3} &= \frac{2}{11} \\ \text{or} \quad \frac{y - 10}{x - 14} &= \frac{2}{11} \end{aligned}\right\}$ These two forms are equivalent and, therefore, either is correct.

4. (a)



Graph (a) shows the "braid" that students would get by graphing each case separately.

(b)



Graph (b) shows a "best" line graph for the data given in this problem.

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5. At load of 0, load is 20.

At load of 300, load is 25.

Slope of "best" line $= \frac{25 - 20}{300 - 0} = \frac{5}{300}$

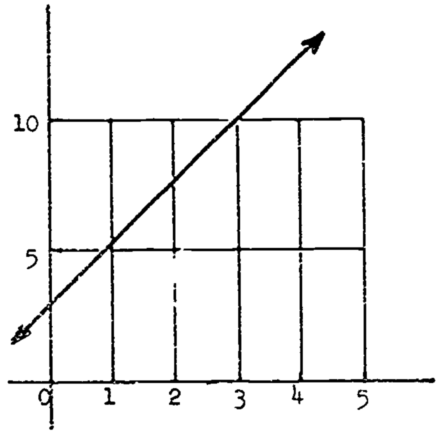
$$\frac{p - 20}{150 - 0} = \frac{1}{60}$$

$$p - 20 = \frac{150}{60} = \frac{5}{2}$$

$$p = 20 + 2.5$$
$$= 22.5 \text{ cm}$$

6. $y = 2x + 4$

x	y
0	4
3	10
1	6
$\frac{1}{2}$	5
4	12



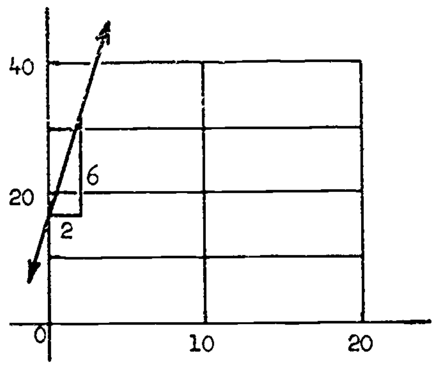
7. $y - 6 = 3(x + 4)$

$y - 6 = 3x + 12$

$y = 3x + 18$

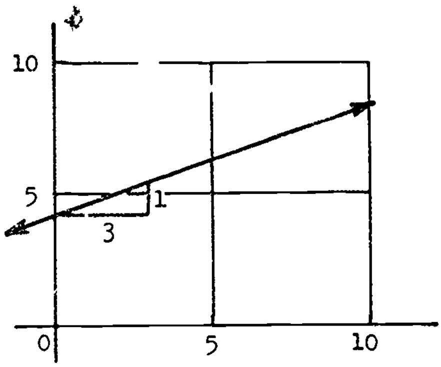
slope: $m = 3$

intercept $(0,18)$



8. $m = \frac{1}{3}$ $(0,4)$

equation: $y = \frac{1}{3}x + 4$



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9. (0,5), (2,7)

Slope: $m = \frac{7 - 5}{2} = \frac{2}{2} = 1$

Equation: $y = x + 5$

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Chapter 3

TRAMPOLINES AND GASES

3.1 Introduction

This section serves to set the stage for a physical experiment about the trampoline. It reminds the student of the experience of bouncing on a trampoline.

3.2 The Trampoline Experiment

This experiment will be performed by the teacher as a class demonstration. You will need to use student help to do the experiment. The equipment needed will be:

- 1 9-inch aluminum pie plate
- 6 15-cent balloons, spherical
- 1 10 × 24-inch sheet bristol board
- 1 pound of modeling clay (for leveling plate)
- 2 glass balls (marbles)
- 1 desk lamp (or slide projector)
- 2 $\frac{5}{8}$-inch nylon bearings (if available)

The procedure for performing the experiment in the student text may be varied very effectively by use of an overhead projector. The projector is laid on the floor and the scale (on a transparency) is placed on the stage of the projector. The images of the bounces are projected on a wide paper tape hung for a screen.

A series of bounces should be made to establish approximately where each bounce number will be found on the scale. It is important that when data is actually recorded, the trials be made on one bounce number at a time.

A sample data sheet is provided in Figure 1 (data for nylon ball shown).

THE TRAMPOLINE

Bounce number (1) n	Height in cm (h) (2) trial 1	(3) trial 2	(4) trial 3	(5) trial 4	Average height (6) h	Corrected height (7) h_n	(8) h_{n+1}	Calculated heights (9) h_{n+1}
0	50.1	(obtained from shadow)			50.1	46.0	40.3	39.6
1	44.5	44.5	44.4	44.3	44.4	40.3	34.2	34.0
2	38.3	38.2	38.2	38.4	38.3	34.2	27.7	29.3
3	31.9	31.8	31.8	31.8	31.8	27.7	23.0	25.2
4	27.3	26.8	27.0	27.2	27.1	23.0	19.7	21.6
5	23.8	23.7	23.9	23.6	23.8	19.7	17.0	18.6
6	21.3	21.1	21.0	21.0	21.1	17.0	15.5	16.0
7	19.2	19.3	19.5	20.3	19.6	15.5	13.0	13.8
8	17.3	16.8	17.3	16.9	17.1	13.0	11.0	11.9
9	15.1	14.9	14.8	15.5	15.1	11.0	9.0	10.1
10	13.8	11.2	13.7	13.8	13.1	9.0		
						obtained by subtracting 4.1 cm		NYLON BALL

$\frac{5}{8}$-inch nylon bearing (diam. 1.4 cm)

light source ≈ 4 meter distant

Figure 1

Bounce number n	THE TRAMPOLINE Height in cm (h) trial 1	trial 2	trial 3	Average height h	Corrected height h_n		
0	50.2			50.2	46.1		
2	45.3	45.4	45.5	45.4	41.3		
4	41.3	41.3	41.2	41.3	37.2		
6	37.2	37.4	37.0	37.2	33.1		
8	32.4	32.4	32.4	32.4	28.3		
10	30.8	31.1	30.6	30.8	26.7		

GLASS
MARBLE

-- DATA DISPLAYED ON (n,h) PLOT --

Figure 2

3.3 Function of Integers

The results of this experiment, when graphed on coordinate paper as in Figure 3, indicate clearly that the points do not lie in a straight line. While this is a perfectly good graph and has an equation, the ordered pairs for the graph are single points and the use of the equation, if the students could find it, is beyond this present level of mathematical knowledge. Certainly it is not a linear function as the chapter heading indicates. The data obtained for this experiment and the graph of the data is included in Figure 1 and Figure 2.

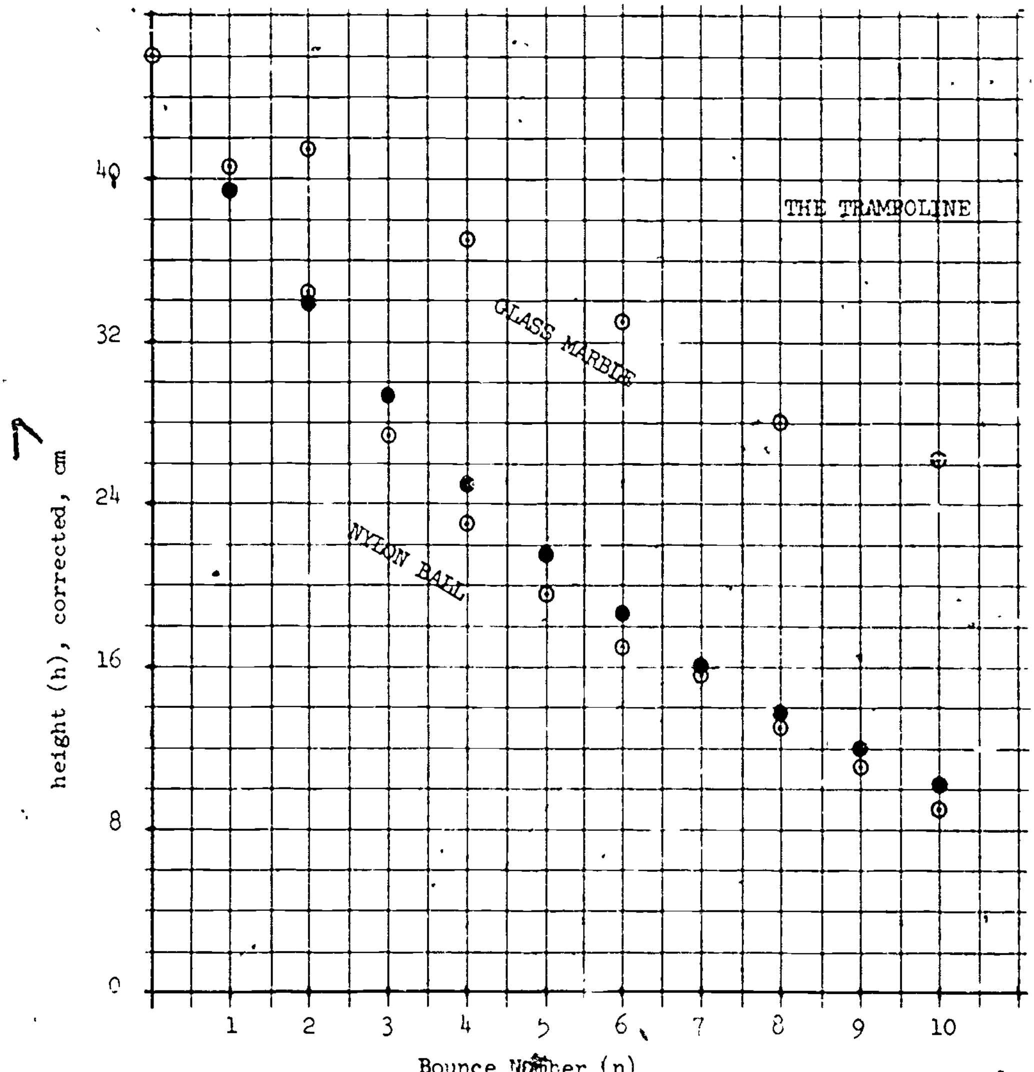


Figure 3

Exercise 1

1. Can your graph of bounce number (n) and bounce height (h) be used to interpolate values of the height for nonintegral values of the bounce number? Explain.

   No. The bounce height occurs once, and once only, for each integral number 0, 1, 2, 3, etc. There is no such thing as as a bounce number $3\frac{1}{3}$.

2. Why can your (n,h) relation be referred to as a function?

   Each number n (0, 1, 2, 3, etc.) gives a unique corresponding height h.

3. Do you think that the (n,h) graph can be extended to find values of the maximum bounce heights for bounce numbers greater than 10? If so, to what value of n would you be willing to go?

   Yes, for greater than 10, but the height will be harder to measure. As n increases, the bounce heights become more difficult to measure. Furthermore, the bounces soon stop!

4. Construct a graph that shows roughly the time of sunset for each day of this week.

   Data for this could be best collected from local newspapers. Almanacs and calendars include general information for some time zones and these times are not usually true locally. Such times for special localities are usually given in T.V. weather broadcasts.

3.4 Mathematical Trampoline Model

At this point we look more closely at the data and try to find some relation which does yield a straight line graph. Fortunately, in this case we succeed quickly.

We suspect that there is a relation between the height to which the ball bounces and the height to which it bounced the time before. With the data already collected we are able to make a table of values, giving for each bounce the height of that bounce and the height of the next bounce. The table is written in the student's text as the ordered pairs

$$(h_0, h_1)$$
$$(h_1, h_2)$$
$$(h_2, h_3)$$
$$\vdots$$
etc.

The corresponding table for the experimental data used in the previous sections is shown in columns (7) and (8) of Figure 1.

These ordered pairs, (h_n, h_{n+1}), were used in plotting the points for the graph in Figure of the students' text. When they use their own data, the graph will be slightly different.

In this case it is also possible to fill in the points on the line by varying the height from which the ball is dropped. The slope must be obtained by measuring distances for the coordinates of two points on the line. For example, when (23, 19.6) and (13, 11.0) are points on the line, the slope is

$$m = \frac{19.6 - 11}{23 - 13} = \frac{8.6}{10} = 0.86$$

and the equation of the line is

$$h_{n+1} = 0.86\, h_n \; .$$

If the equation is written in the form

$$h_{n+1} = mh_n$$

then

$$m = \frac{h_{n+1}}{h_n} \; .$$

This will enable us to determine all of the bounce heights in terms of the bounce number and the height of the previous bounce -- or more simply, in terms of the number n and the original height from which the ball was dropped. The data table could have been written in the form

n	h
0	$m \cdot 50$
1	$m(m \cdot 50) = m^2 \cdot 50$
2	$m^3 50$
3	$m^4 50$
.	.
.	.
.	.
n	$m^{n+1} 50$

Figure 4

The result of this is the equation given in the students' text,

$$h_{n+1} = m^{n+1} h_0 .$$

Since m is constant (as shown by the straight line graph, Figure 5) for a given trampoline system, the variables are n and h_0. and the equation is one with a variable in the exponent. It is certainly not linear and is restricted to a particular domain, but we have been able to find the equation by the use of our knowledge of linear functions.

Figure 5 shows the graph of the linear function whose equation is

$$h_{n+1} = 0.86h_n .$$

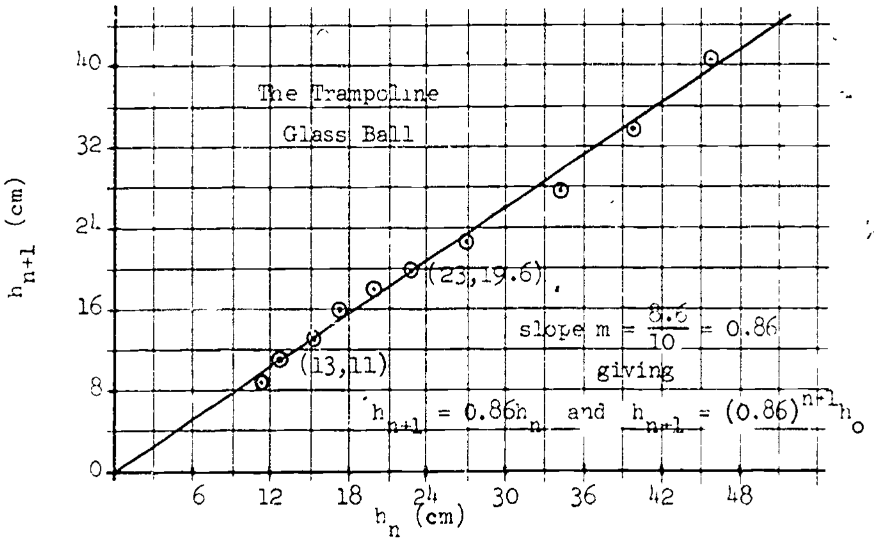


Figure 5

You might wish to have an easier way to calculate the height of the ball after the n^{th} bounce without extensive computation (i.e., raising decimal fractions to integral powers). The following graphical procedure will serve this purpose.

You might wish to present the following graphical procedure for computing the height of the ball after the n^{th} bounce to your students. The student has prepared a graph of h_{n+1} vs. h_n, and has drawn the best fitting line on this graph. The slope of this line is called m. Notice also that this line has been drawn so that it passes through the origin. Have the student add another line to this graph. This new line is also to be through the origin, but is to have slope one.

On the horizontal axis, find the coordinate which represents the original height of the ball. Find the point on the line of slope m which has this horizontal coordinate. Suppose that the original height was 50 cm. Then this point will have horizontal coordinate 50 and vertical coordinate 50 m. Following a horizontal line through this point to the line with slope one, we obtain the point (50m,50m). Following a vertical line down from this point to the data line gives the point (50m,$50m^2$). Continuing in this way, the staircase-shaped line shown in the illustration below leads from point to point on the line with slope m. The successive points have horizontal coordinates representing the height of the ball after the appropriate number of bounces. (See Figure 6.)

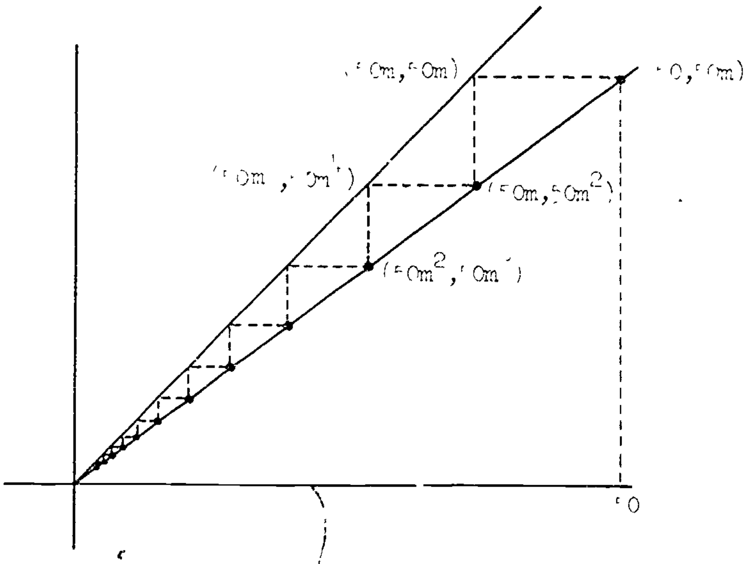


Figure 6

3.5 Experimental Extension

If a nylon bearing is not available, then a wood or cork ball might be substituted. Also, one can dispense with the trampoline and use the ball from a roll-on-deodorant jar and bounce it off the floor.

Exercise 2

1. Referring to your graph of the (h_n, h_{n+1}) relation, what is the domain and range of the experimental data? What restrictions, if any, would you place upon the domain and range of the mathematical equation found to represent the line?

   This refers to the students' counterpart of Figure 5 in the text. The domain (h_n) will be approximately $6 \leq h_n \leq 50$ and the range (h_{n+1}) $5 \leq h_{n+1} \leq 40$. (The students' measurements may vary considerably.) The domain and range of the mathematical equation representing this data cannot safely be extended too far from the experimental data.

2. Suppose that

$$h_{n+1} = (0.5)^{n+1} h_0 .$$

Sketch to the same scale a series of (n,h) points for $h_0 = 10$, 50 and 100.

For $h_0 = 10$

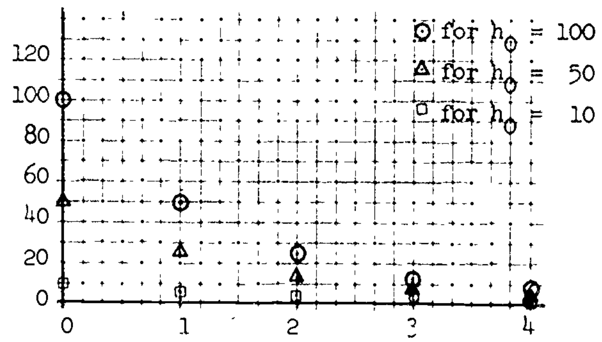
n	h
0	10
1	5
2	$2\frac{1}{2}$
3	$1\frac{1}{4}$
4	$\frac{5}{8}$

For $h_0 = 50$

n	h
0	50
1	25
2	12.5
3	6.25
4	$3\frac{1}{8}$

For $h_0 = 100$

n	h
0	100
1	50
2	25
3	12.5
4	6.25



3. Suppose that

$$h_{n+1} = m^{n+1} \cdot 100 .$$

Sketch to the same scale a series of (n,h) points for $m = 0.3$, 0.6 and 0.9.

For $m = 0.3$

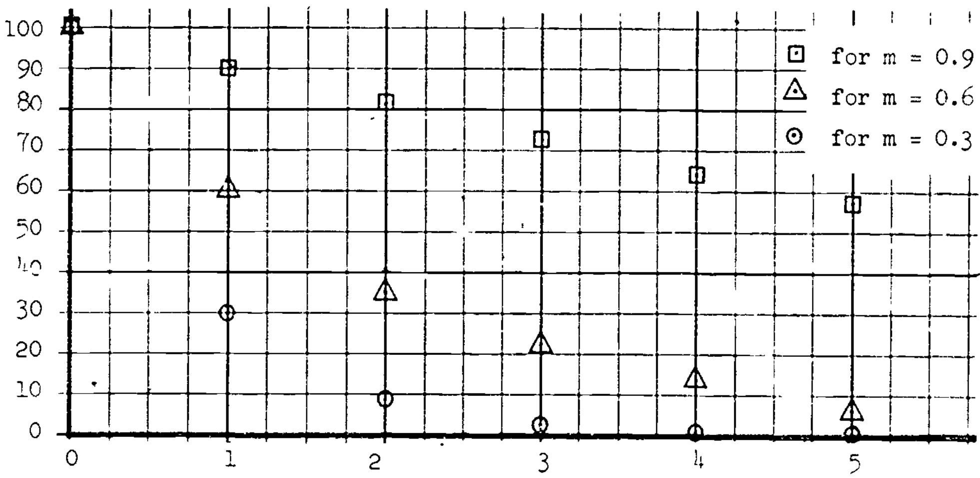
n	h
0	100
1	30
2	9
3	2.7
4	.81
5	.24

For $m = 0.6$

n	h
0	100
1	60
2	36
3	21.6
4	13
5	7

For $m = 0.9$

n	h
0	100
1	90
2	81
3	72.9
4	64.6
5	58.1



4. Make a possible interpretation of the significance of the equation

$$h^{*}_{n+1} = (0.5)^{n+1} \cdot 100$$

for the case $n = -1$.

The point of release is 100 for bounce number 0.

5. Why did the domain of the relation

$$h_{n+1} = m^{n+1} h_0$$

include the value $n = 9$ and not $n = 10$?

If $n = 10$, then $n+1 = 11$, and there was no bounce number 11.

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6. What is the physical unit of the quantity "m" in the equation

$$h_{n+1} = m^{n+1}h_0 \ ?$$

None. "m" is a ratio or quotient.

7. Do you think it would be possible to find a value of m greater than or equal to 1 ? Explain.

No. This would imply the height of bounce is higher than the point of release.

3.6 <u>Gay-Lussac's Law Experiment</u>

This experiment will deal with the change in pressure of an enclosed gas (ordinary air) vs the temperature. However, we shall be interested in this experiment because there is some physical meaning to the extension of the domain (the temperature) beyond the experimental data collected and the accompanying mathematical concepts of graphical and algebraic translation of the coordinate axes.

Because of the expense of the equipment and the fact that hot water is used, this experiment will be done by the teacher as a class demonstration. You will need to have a few students assist you in the experiment. The equipment needed is:

A Gay-Lussac's Law (also called Charles' Law) apparatus
(c[illegible] study - cenco No. 76525)
Centigrade thermometer
Vessel for water
A source of hot water
Ice
Heater to raise water to 100 degrees C (optional)
Ring stand to hold apparatus (optional)

Data for the table of values (temperature, pressure) will be read by the experimenters and should be written on the chalkboard and copied by each student in the class. The table of values found in this experiment is given on the following page. (See Figure 7.)

Each student will then graph this data on a coordinate system using the appropriate scales. He should then be able to draw his best straight line as has been done in the other experiments.

The students should also be able to write the equation of the line

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quickly by the use of the point-slope form or perhaps by the slope-intercept form if you were able to reduce the temperature to 0 degrees C in the experiment.

For the data given above, the graph appears in Figure 8.

Gay-Lussac's Law		
Temp. (^{o}C)	Pressure (lbs/in^2)	
99.4	18.0	boiling water
85.5	17.6	
75.5	17.2	
65.5	16.4	
55.5	16.0	
45.5	15.6	
35.5	15.0	
25.5	14.6	
21.0	14.4	
10.5	13.8	
1.2	13.4	ice mixture
gauge tapped gently before each reading.		

Figure 7

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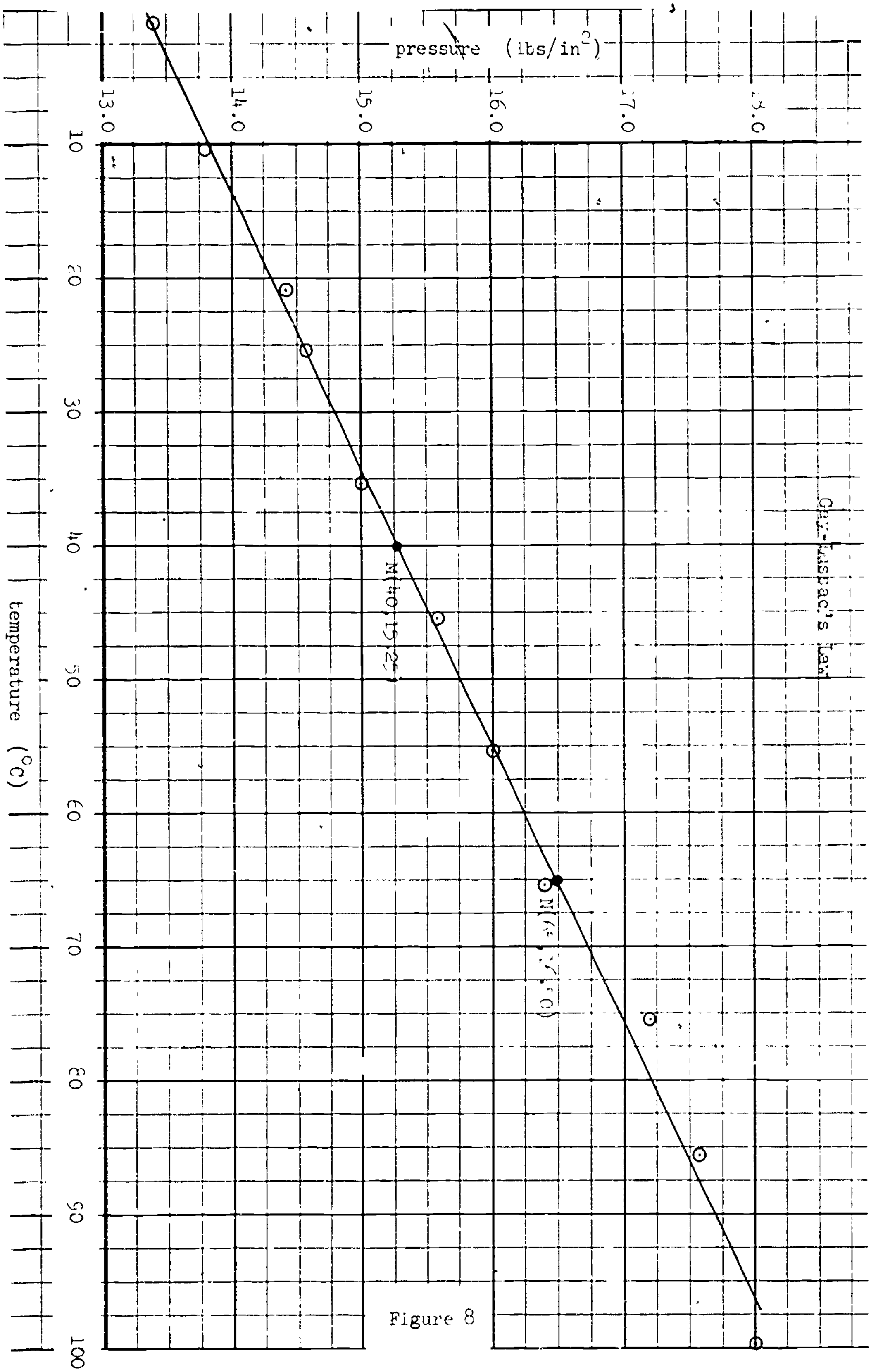

pressure (lbs/in²)
13.0
14.0
15.0
16.0
17.0
18.0
temperature (°C)
10
20
30
40
50
60
70
80
90
100
Gay-Lussac's Law
M(40,15.25)

Figure 8

A best straight line is drawn and its slope is found by the use of the points M and N to be

$$m = \frac{16.50 - 15.25}{65 - 40} = \frac{1.25}{25} = 0.05.$$

If (C,P) represents any arbitrary point on the line, we may use the point M (40,15.25) to write the equation of the line

$$P - 15.25 = 0.05\ (C - 40)$$
$$P = 0.05C + 13.25\ .$$

No exercise is given at this point because we are more interested in the possibility of extending the domain and range of the function.

3.7 Extending the Temperature Domain

The idea of interpolation is brought up again here. At the end of the section problems on this and the preceding experiment are included. The idea of extrapolation is emphasized, though still with some caution. There are apparent physical limits in the other experiments which are more easily recognized. These limits must be considered by the students in solving the exercises.

Since it makes more sense to extend the domain and hence the graph of Gay-Lussac's Law to the left, we do so and are able to arrive at a temperature for zero pressure which is approximately the absolute zero which scientists know is slightly lower than -273 degrees C. Perhaps the students have also heard of this temperature.

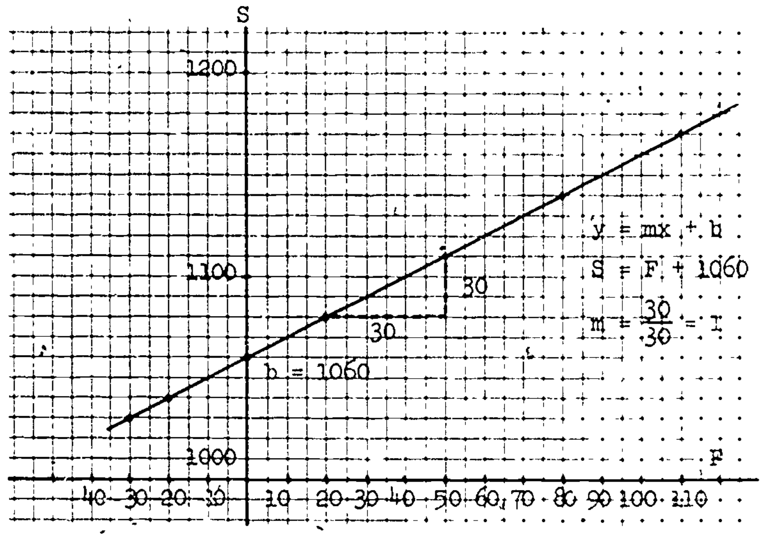
Exercise 3

1. The table below shows the speed of sound in air at various Fahrenheit temperatures. The absolute zero of temperature on the Fahrenheit scale is $-460^{\circ}F$.

Temp. ($^{\circ}$F)	-30	-20	0	20	50	80	110
Speed (ft/sec)	1030	1040	1060	1080	1110	1140	1170

(a) Draw a graph showing the relation between F and speed of sound (S). Make temperature values the domain and let the origin represent 1000 on the vertical axis.

(b) Write the equation for the relation between F and S.

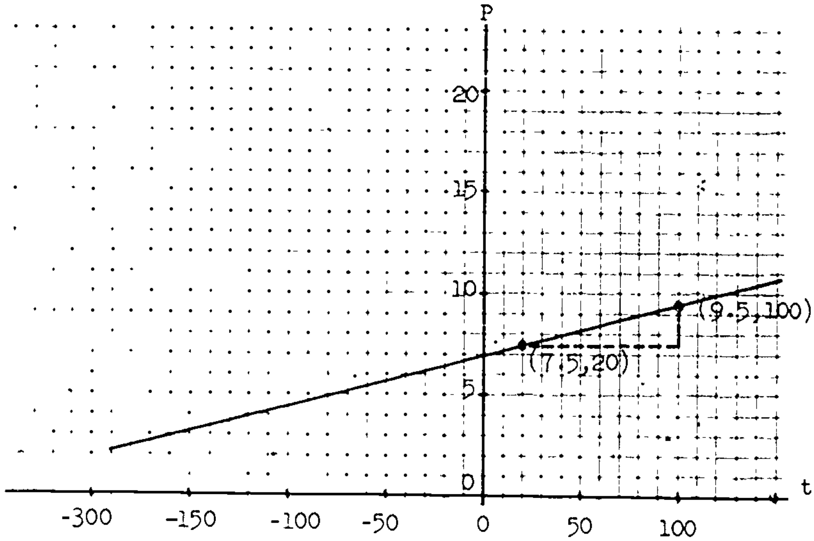


2. The relation between Centigrade and Fahrenheit temperatures is expressed in the equation $C = \frac{5}{9}(F-32)$.
Write the equation obtained by reversing the variables.

$$F = \frac{9}{5}C + 32 .$$

3. In an experiment on Gay-Lussac's Law, a student found that the pressure of the gas was 7.5 lb/sq in at $20^{\circ}C$, and 9.5 lb/sq in at $100^{\circ}C$.

(a) Graph the relation



(b) Write the equation representing the relation between the pressure (P) and the Centigrade temperature (t).

Since $m = \frac{2}{80} = \frac{1}{40}$; $b = 7$ and $P = mt + b$

$$\therefore P = \frac{1}{40}t + 7 .$$

(c) At what temperature would the pressure of the gas be 8.2 lb/sq in?

$48^{\circ}C$

(d) What would be the pressure of the gas at $50^{\circ}C$?

8.25 lb/sq in

3.8 <u>Graphical Translation of Coordinate Axes</u>

Translating axes by the use of the sheet of frosted acetate (or other substitutes) is not difficult and will often help the student to simplify the equation of the line and the orientation of the graph. Each student should have his own sheet of acetate. Before the acetate sheets are passed out, draw a set of coordinate axes in the middle of each sheet. Use a heavy ball-point pen. Do not write any number or scales on the sheet. Students will write these with a pencil since they can then be easily erased.

While it is possible to position the new axes in any manner whatsoever, (i.e., they can be translated and rotated) translation horizontally and vertically will be all we do here.

The amount and direction of translation is purely arbitrary. But in graphs based on science experiments, the physical situation usually suggests a "natural" location for the new axes. For example, when dealing with Gay-Lussac's Law it will be the point at which the pressure is zero. The only way the student will be able to approximate this point will be to extend his graph to the left until it crosses the temperature-axis (not included in his original graph). If this point of zero pressure is near to -273 degrees on the C-axis (between -260 degrees C and -285 degrees C) it will be close enough. The equation in the new coordinate system will obviously be in the form

$$Y = mX \qquad \text{(See Figure 9.)}$$

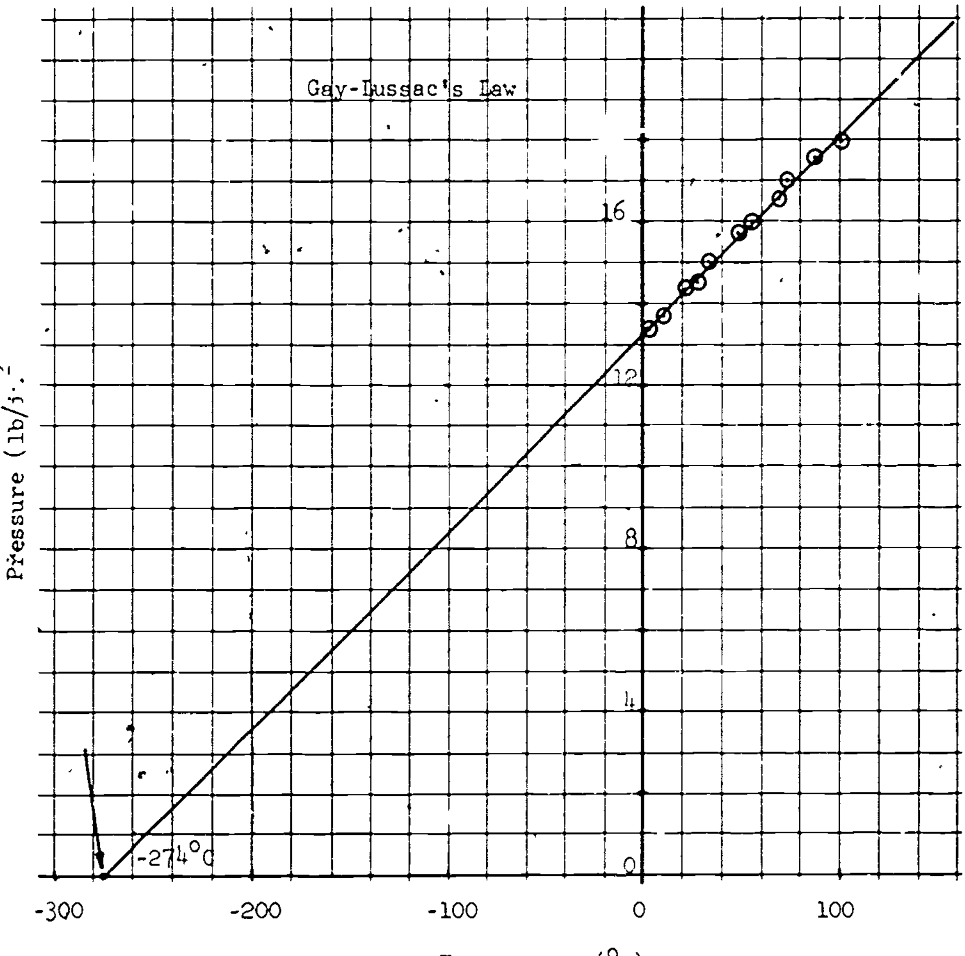


Figure 9

Exercise 4

1. Find your load-position graph that you drew in the Loaded Beam Experiment. Using a sheet of frosted acetate that carries coordinate axes X and Y, translate the origin on the overlay to the y-intercept on your graph. What is the equation of your "test" straight line with respect to the shifted axes?

   $Y = 0.015X$

   (using data presented in Teacher's Commentary - Chapter _)

2. How could you perform the Loaded Beam Experiment to obtain the equation found in Exercise 1 directly?

   Have the meter stick placed so the end of the beam coincides with 0 on the stick.

3. Draw the line in the first quadrant that contains the point (2,3) and whose slope is $\frac{1}{2}$. Use your plastic overlay to obtain the new equations of this line when the origin is shifted

   (a) to the y-intercept;

   $y = \frac{1}{2}x$

   (b) to the left 3 units;

   $y = \frac{1}{2}x + 3.5$

   (c) to the right 4 units and up 3 units;

   $y = \frac{1}{2}x + 2$.

4. Draw the line in the first quadrant which contains the points (1,7) and (7,5). Use your plastic overlay to obtain the new equations of this line when the origin is shifted

   (a) to the intercept;

   $y = -\frac{1}{3}$

   (b) to the y-intercept;

   $y = -\frac{1}{3}x$

   (c) to the point (4,6);

   $y = -\frac{1}{3}x$.

3.9 Algebraic Translation of Coordinate Axes

This section is purely a mathematical discussion of translation of the axes. Though we did write the equations of the line representing Gay-Lussac's Law at the end of the preceding paragraph, this cannot always be done as easily, especially when the graph represents curved lines.

The explanations of the algebraic translation of axes in the students' text is clear and needs no elaboration here. You may feel, however, that more drill similar to Exercise 4 will be needed. All you will need to provide this drill is to give the class equations of the sort $3x - 2y = 7$ and ask them to translate the origin to various points like the y-intercept, the point $(5,4)$ or $(0,6)$, and so on.

To translate the axes of this equation to the point $(5,4)$ write it in the form

$$y = \frac{3}{2}x - \frac{7}{2}$$

or

$$y - 0 = \frac{3}{2}\left(x + \left(-\frac{7}{3}\right)\right).$$

It is now in the point-slope form and you may use equation (2)

$$Y + (0) + K = m\left(X + \left(-\frac{7}{3}\right) + h\right)$$

where h and K are 5 and 4 respectively. This becomes

$$Y + 0 + 4 = \frac{3}{2}\left(X + \left(-\frac{7}{3}\right) + 5\right)$$

$$Y + 4 = \frac{3}{2}\left(X + \frac{8}{3}\right)$$

or

$$Y = \frac{3}{2}X .$$

Since the point $(5,4)$ is on the line, you would expect to get equations in the form

$$Y = mX .$$

If, on the other hand, you want to translate to the point $(5,-4)$, it is done in this manner:

$$Y + (0) + (-4) = \frac{3}{2}\left(X + \left(-\frac{7}{3}\right) + 5\right)$$

$$Y - 4 = \frac{3}{2}\left(X + \frac{8}{3}\right)$$

$$Y - 4 = \frac{3}{2}(X) + 4$$

$$Y = \frac{3}{2}X + 8 .$$

Exercise 5

1. When we extended the temperature domain for the Gay-Lussac's Law Experiment, we found that the graph intercepted the temperature axis near the $(-273,0)$ point. Algebraically translate the origin of your graph to this intercept. Write the new equation of the line. What are the new units of temperature, pressure and the slope of the line?

   Using the equation $P - 15.25 = 0.05\ (C - 40)$, we translate the axes to $(-273,0)$

   $$y + 0 = 0.05\ (x - 233)\ .$$

   The new units of temperature are degrees C, of pressure pounds per square inch. The slope of the line is the same, since this is not affected by translation of the axes.

2. Draw the line in the first quadrant that contains the point $(2,3)$ and whose slope is $\frac{1}{2}$. Write the equation of this line in point-slope form. Obtain the equation of this line algebraically when the origin has been translated

   $$y - 3 = \frac{1}{2}\ (x - 2)$$

   (a) to the y-intercept

   $$y = \frac{1}{2}x$$

   (b) to the left 3 units

   $$y - 3 = \frac{1}{2}\ (x - 5)$$

   (c) to the right 4 units and up 3 units

   $$y = \frac{1}{2}\ (x + 2)$$

   Compare your results to those obtained graphically in Exercise 3 of the previous section.

3. Draw the line in the first quadrant which contains the points $(1,7)$ and $(7,5)$. Write the equation of this line in point-slope form. Obtain the equation of this line algebraically when the origin has been translated

   (a) to the x-intercept $\qquad y + \frac{1}{3} = \frac{1}{3}\ (x - 1)$

   (b) to the y-intercept $\qquad y - 7 = -\frac{1}{3}\ (x + 21)$

   (c) to the point $(4,6)$ $\qquad y - 1 = -\frac{1}{3}\ (x + 3)$

   Compare your results to those obtained graphically in Exercise 4 of the previous section.

Sample Test Items

True-False Items

1. All measurements are approximate.
2. Lines drawn on the same coordinate paper and found to be parallel have the same slope.
3. In the "temperature-pressure" experiment, the pressure of the gas decreased as the temperature was increasing.
4. Not all gases behave in the manner described by your graph when you extrapolated to lower temperatures.
5. When you translate axes you do <u>not</u> change the graph, but merely the mathematical description.

Problems

1. In the trampoline experiment, we found the slope $m = \frac{h_{n+1}}{h_n}$, a ratio between two successive bounce heights.

   (a) Find m if $h_4 = 36$ and $h_5 = 27$.

   (b) Find m if $h_7 = 18$ and $h_8 = 12$.

   (c) Were these measurements taken on the same type of ball?

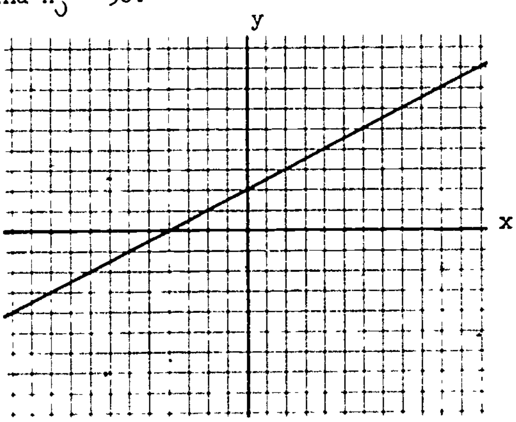
2. In the trampoline experiment, we discovered the equation $h_{n+1} = m^{n+1}h_0$.

   (a) Find h_{n+1}, if $n = 3$, $m = \frac{1}{2}$ and $h_0 = 64$.

   (b) Find h_{n+1}, if $n = 2$, $m = \frac{3}{4}$ and $h_0 = 96$.

3. (a) In which direction must you translate the graph to change the y-intercept to zero?

   (b) How much must it be translated?



4. Examine why the value of m in the Trampoline Experiment does not exceed 1.

Answers to Sample Test Items

True-False

1. T
2. T
3. F
4. T
5. T

Problems

1. (a) $m = \frac{3}{4}$

   (b) $m = \frac{2}{3}$

   (c) No

2. (a) $h_4 = 4$

   (b) $h_3 = 40.5$

3. (a) Down

   (b) 2 units

4. The ball would bounce higher than the point from which it was released.

Appendix A

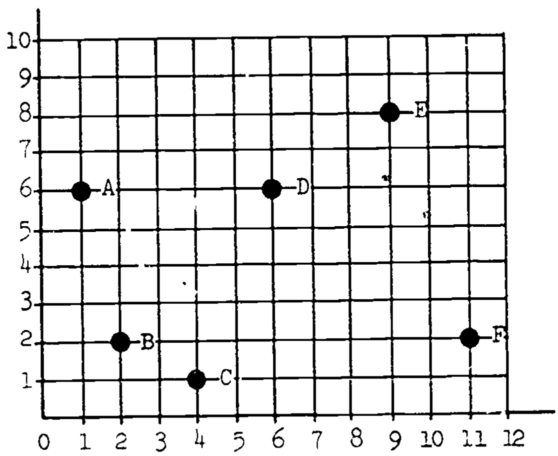
GRAPHING EXPERIMENTAL DATA

A knowledge of the meaning of ordered pairs and how to plot ordered pairs on the coordinate plane is needed before beginning Chapter 2. Appendix A gives a brief treatment of the subject and introduces the terminology necessary for the student to understand Chapters 2 and 3.

If your students already have a knowledge of plotting ordered pairs you may want to spend one class period on this appendix as a review. The problems provide an introduction to the type of material which will be covered in the last two chapters.

Exercise 1

1. Write the ordered pairs of numbers which are associated with the points A through F in the figure below.

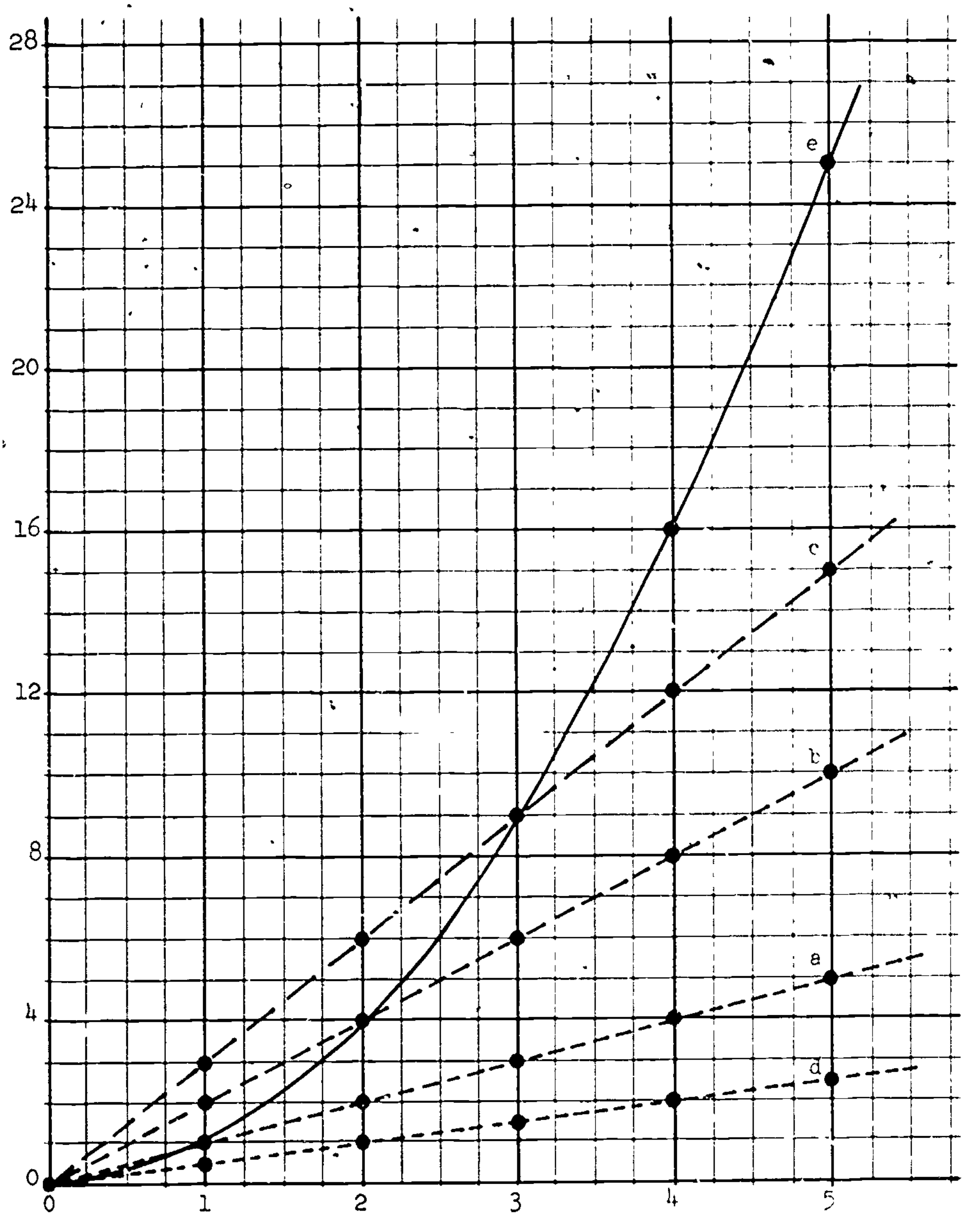


2. Graph the following sets of ordered pairs on the same sheet of graph paper.

(a) (0,0), (1,1), (2,2), (3,3), (4,4), (5,5)
(b) (0,0), (1,2), (2,4), (3,6), (4,8), (5,10)
(c) (0,0), (1,3), (2,6), (3,9), (4,12), (5,15)
(d) (0,0), $(1,\frac{1}{2})$, (2,1), $(3,1\frac{1}{2})$, (4,2), $(5,2\frac{1}{2})$
(e) (0,0), (1,1), (2,4), (3,9), (4,16), (5,25)

NOTE: It will probably be helpful to begin this problem as a classroom assignment. Notice that in all five sets the first element of any ordered pair never gets larger than 5 while, in part (e), the second element of the last ordered pair is 25. The horizontal and vertical axes <u>do not</u> have to have the same scales and a graph should be made so as to "fill" the paper. This point will be emphasized in Chapter 2.

The text makes no mention of connecting the points. However it will be helpful to have the students connect the points with dotted lines (or a dotted curve for part (e)) simply as a means of "keeping the set together". Chapters 2 and 3 will discuss continuity and when points may be connected.



ERIC

3. Make a set of at least five ordered pairs to satisfy the following conditions.

The sets below are representative of answers the students should have.

(a) The ordered pairs for which the vertical coordinate is 6 times the horizontal coordinate.

$(y = 6x) : \{(0,0), (\frac{1}{3},2), (\frac{1}{2},3), (1,6), (2,12)\}$

(b) The ordered pairs for which the vertical coordinate is 3 times the horizontal coordinate.

$(y = 3x) : \{(0,0), (\frac{1}{3},1), (1,3), (2,6), (3,9)\}$

(c) The ordered pair for which the vertical coordinate is 2 more than twice the horizontal coordinate.

$(y = 2x + 2) : \{(0,2), (\frac{1}{2},3), (1,4), (2,6), (3,8)\}$

(d) The ordered pairs for which the vertical coordinate is the square root of the horizontal coordinate.

$(y = \sqrt{x}) : \{(0,0), (1,1), (4,2), (9,3), (16,4)\}$

(e) The ordered pairs for which the vertical coordinate is the cube of the horizontal coordinate.

$(y = x^3) : \{(0,0), (1,1), (2,8), (3,27), (4,64)\}$

Two additional problems which could be given as a classroom exercise follow:

(a) Graph the set of ordered pairs which were formed for Problem 2.

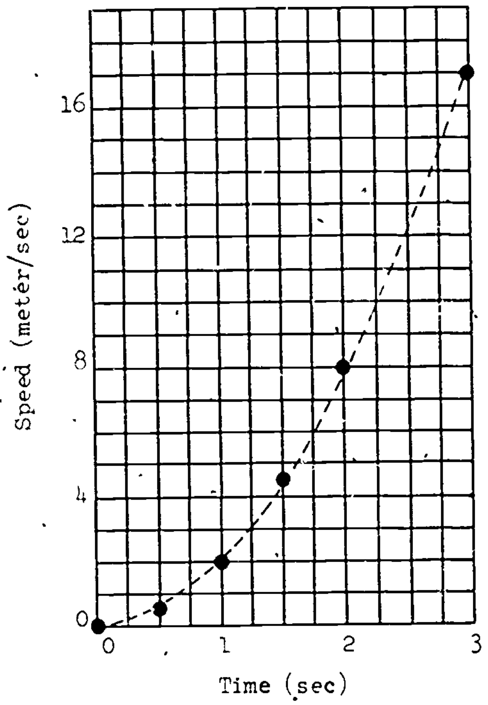
(b) Write statements (similar to those in Problem 2) to describe the sets of ordered pairs in Problem 1.

4. Make a graph of the data recorded in each of the tables below. In each case, the top row indicates the horizontal coordinates and the bottom row the vertical coordinates. Be sure to label the axes correctly (refer to Figure 5 in text).

ERIC

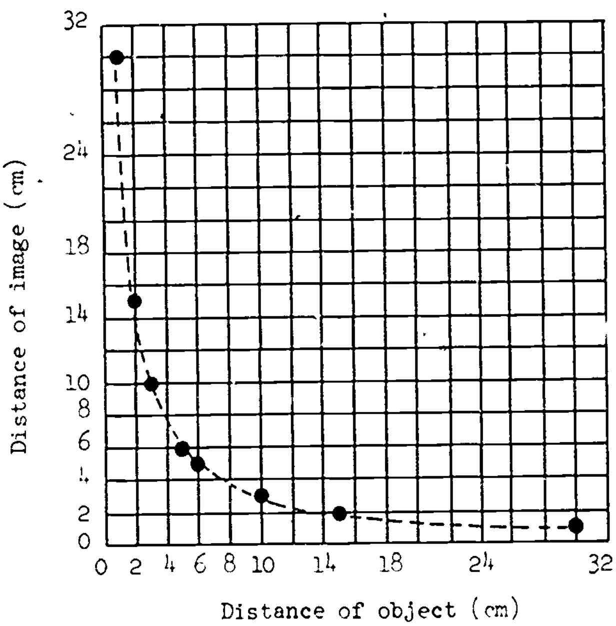
(a)

Time (sec)	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	3
Speed (meters/sec)	0	$\frac{1}{2}$	2	$4\frac{1}{2}$	8	18



(b)

Distance of object (cm)	30	15	10	6	5	3	2	1
Distance of image (cm)	1	2	3	5	6	10	15	30



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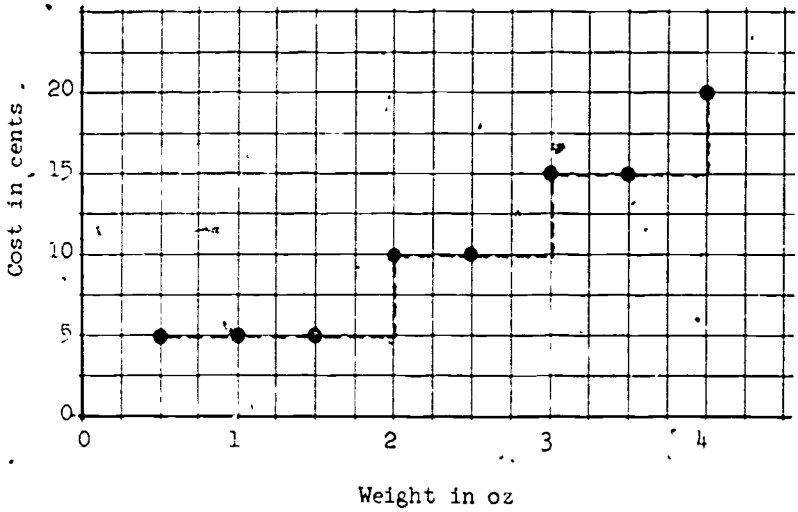
(c)

Weight in oz	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4
Cost in cents	5	5	5	10	10	15	15	20



1 . 3

Appendix B

SCIENTIFIC NOTATION

Scientific notation is not used as such in the text; however some familiarity with exponents is assumed in Chapter 3. Students at this level probably have studied scientific notation and a brief review may be sufficient. This appendix is designed for those classes or individual students who have not previously been exposed to this type of material. The text should be taught for familiarity with the procedures and not necessarily for complete mastery of the subject.

Exercise 1

1. For each of the following, indicate the base and the exponent.

	Base	Exponent		Base	Exponent
(a) 6^3	6	3	(d) 9^2	9	2
(b) 10^5	10	5	(e) x^2	x	2
(c) 5^8	5	8	(f) x^5	x	5

2. Using exponents, write each of the following in briefer form.

(a) $3 \times 3 \times 3 \times 3$ $= 3^4$

(b) $10 \times 10 \times 10$ $= 10^3$

(c) $3 \times 3 \times 3 \times 5 \times 5$ $= 3^3 \times 5^2$

(d) $5 \times 3 \times 2 \times 2 \times 3 \times 5$ $= 2^2 \times 3^2 \times 5^2$

(e) $1.2 \times 10 \times 10 \times 10$ $= 1.2 \times 10^3$

3. What is the value of each of the following?

(a) 3^4 $= 3 \times 3 \times 3 \times 3 = 81$

(b) 2^3 $= 2 \times 2 \times 2 = 8$

(c) 9^2 $= 9 \times 9 = 81$

(d) 5^4 $= 5 \times 5 \times 5 \times 5 = 625$

(e) 10^6 $= 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1,000,000$

(f) $3^2 + 2^3$ $= (3 \times 3) + (2 \times 2 \times 2) = 9 + 8 = 17$

(g) $3^3 + 2^2$ $= (3 \times 3 \times 3) + (2 \times 2) = 27 + 4 = 31$

In Problems 4-7, tell which statements are true and which are false.

4. $2^3 \times 3^3 = 6^3$

$$2^3 \times 3^3 = (2 \times 2 \times 2) \times (3 \times 3 \times 3) = 8 \times 27 = 216$$

$$6^3 = 6 \times 6 \times 6 = 216$$

$216 = 216$ The statement is true.

5. $2^3 \times 2^3 = 2^6$

$$2^3 \times 2^3 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) = 8 \times 8 = 64$$

$$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$$

$64 = 64$ The statement is true.

6. $3 \times 3^3 = 9^3$

$$3 \times 3^3 = 3 \times (3 \times 3 \times 3) = 3 \times 27 = 81$$

$$9^3 = 9 \times 9 \times 9 = 729$$

$81 \neq 729$ The statement is false.

7. $2^5 - 2^3 = 2^2$

$$2^5 - 2^3 = (2 \times 2 \times 2 \times 2 \times 2) - (2 \times 2 \times 2) = 32 - 8 = 24$$

$$2^2 = 2 \times 2 = 4$$

$24 \neq 4$. The statement is false.

Exercise 2

1. Perform the indicated multiplications mentally and write your answers.

(a) $259.4 \times 10^{-4} = 0.02594$

(b) $3.258 \times 10^{2} = 325.8$

(c) $.023 \times 10^{3} = 23$

(d) $35.68 \times 10^{-1} = 3.568$

(e) $358.2 \times 10^{-3} = 0.3582$

(f) $151 \times 10^{-4} = 0.0151$

(g) $.0031 \times 10^{5} = 310$

(h) $29.35 \times 10^{-2} = 0.2935$

(i) $3.05 \times 10^{-6} = 0.00000305$

(j) $3.05 \times 10^{6} = 3,050,000$

2. Express these measurements in scientific notation.

(a) There are more than 4,500,000 [4.5×10^6] red corpuscles per cubic mm of blood.

(b) and (c) If a given sample of material contains 2,000,000 [2×10^6] atoms of U^{238} in 1964, this same sample will contain 250,000 [2.5×10^5] atoms of U^{238} in the year 13,500,001,964. (Write the number of atoms in scientific notation.)

(d) The distance to the sun is 150,000,000 [1.5×10^8] km .

Appendix C

METRIC SYSTEM

The metric system is used for measurements throughout the text. Conversion of units is not called for, but the authors feel that some time spent in a study of the metric system and conversion of units will be valuable.

Exercise 1

1.

Conversion Factors
1 m = 100 cm
1 m = 10 dm
1 dm = 10 cm

100 m = 100(10 dm) = 1000 dm

100 m = 100(100 cm) = 10,000 cm or 1000 dm = 1000(10 cm) = 10,000 cm

100 m = 1000 dm = 10,000 cm

2.

Conversion Factors
1000 mg = 1 g
1000 g = 1 kg

$37.2 \text{ mg} = 37.2(\frac{1}{1000} \text{ g}) = 0.0372 \text{ g}$

$0.0372 \text{ g} = 0.0372 (\frac{1}{1000} \text{ kg}) = 0.0000372 \text{ kg}$

$37.2 \text{ mg} = 3.72 \times 10^{-2} \text{ g} = 3.72 \times 10^{-5} \text{ kg}$

3.

Conversion Factor
1 liter = 1000 ml

281 liters = 281 (1000 ml) = 281,000 ml

4.

Conversion Factors
10 mm = 1 cm
100 cm = 1 m
1000 mm = 1 m

1285 cm = 1285 (10 mm) = 12,850 mm

$1285 \text{ cm} = 1285 (\frac{1}{100} \text{ m}) = 12.85 \text{ m}$

1285 cm = 12,850 mm = 12.85 m

5.

Conversion Factors
1000 mg = 1 g
10 g = 1 dg

0.155 g = 0.155 (1000 mg) = 155 mg

$0.155 \text{ g} = 0.155 (\frac{1}{10} \text{ dg}) = 0.0155 \text{ dg}$

0.155 g = 155 mg = 0.0155 dg

6.

Conversion Factor
1000 kilowatts = 1 watt

$500 \text{ kilowatts} = 500 (\frac{1}{1000} \text{ watt}) = 0.5 \text{ watt}$

7. Conversion Factor
10 dl = 1 liter

900 dl = 900 $(\frac{1}{10}$ liter$)$ = 90 liters

8. Conversion Factor
1000 cycles
= 1 kilocycle

The frequency of radio station WICH is 1340 kilocycles. Express the frequency in cycles.

1340 kc = 1340 (1000 cycles) = 1,340,000 cycles

9. Conversion Factors
10^6 microseconds
= 1 second
60 seconds
= 1 minute

55 minutes = 55 (60 seconds) = 3300 seconds
3300 seconds = 3300 (10^6 microseconds)
= 3.3×10^9 microseconds

Exercise 2

1. Conversior Factors
12 in = 1 ft
2.54 cm = 1 in

3 ft = 3 (12 in) = 36 in
36 in = 36 (2.54 cm) = 914.4 cm

2. Conversion Factors
1 liter = 1.05 qt
4 qt = 1 gal

114 liters = 114 (1.05 qt) = 119.7 qt
119.7 qt = 119.7$(\frac{1}{4}$ gal$)$ = 29.9 gal

3. Conversion Factors
100 cm = 1 m
2.54 cm = 1 in
36 in = 1 yd

27 meters = 27 (100 cm) = 2700 cm
2700 cm = 2700 $(\frac{1}{2.54}$ in$)$ = 1063 in
1063 in = 1063 $(\frac{1}{36}$ yd$)$ = 29.5 yd

4. Conversion Factor
1 ml = 1 cubic centimeter

428 ml = 428 cc

5. Conversion Factors
1 ft = 12 in
2.54 cm = 1 in
100 cm = 1 m

6.5 ft = 6.5 (12 in) = 780 in
780 in = 780 (2.54 cm) = 1980 cm
1980 cm = 1980 $(\frac{1}{100}$ m$)$ = 19.8 m

6. Conversion Factor
454 g = 1 lb

1.5 lb = 1.5 (454 g) = 681 g

7. $\boxed{\begin{array}{c}\text{Conversion Factor}\\ 454\text{ g} = 1\text{ lb}\end{array}}$

$7 \text{ kg} = 7000 \text{ g} = 7000 \left(\frac{1}{454} \text{ lb}\right) = 15.4 \text{ lb}$

8. $\boxed{\begin{array}{c}\text{Conversion Factor}\\ 1\text{ liter} = 1.05\text{ qt}\end{array}}$

$1 \text{ qt} = \frac{1}{1.05} \text{ liter} = 0.95 \text{ liter}$

9. $\boxed{\begin{array}{c}\text{Conversion Factors}\\ 39.37\text{ in} = 1\text{ m}\\ 36\text{ in} = 1\text{ yd}\end{array}}$

$1 \text{ yd} = 36 \text{ in} = 36 \left(\frac{1}{39.37} \text{ m}\right) = 0.91 \text{ m}$

10. $\boxed{\begin{array}{c}\text{Conversion Factor}\\ 454\text{ g} = 1\text{ lb}\end{array}}$

$1 \text{ kg} = 1000 \text{ g} = 1000 \left(\frac{1}{454} \text{ lb}\right) = 2.2 \text{ lb}$